

CONTENTS.

PART I. *Containing the Analysis of Determinate Quantities.*

SECTION I.

Of the Different Methods of calculating Simple Quantities.

	Page
Chap. I. OF Mathematics in general - - -	1
II. Explanation of the signs + <i>plus</i> and - <i>minus</i> -	3
III. Of the Multiplication of Simple Quantities -	6
IV. Of the nature of whole Numbers, or Integers with respect to their Factors - - -	10
V. Of the Division of Simple Quantities - -	13
VI. Of the properties of Integers, with respect to their Divisors - - - -	16
VII. Of Fractions in general - - - -	20
VIII. Of the Properties of Fractions - -	24
IX. Of the Addition and Subtraction of Fractions -	27
X. Of the Multiplication and Division of Fractions	30
XI. Of Square Numbers - - - -	36
XII. Of Square Roots, and of Irrational Numbers resulting from them - - - -	38
XIII. Of Impossible, or Imaginary Quantities, which arise from the same source - - -	42
XIV. Of Cubic Numbers - - - -	45
XV. Of Cube Roots, and of Irrational Numbers resulting from them - - -	46
XVI. Of Powers in general - - - -	48
XVII. Of the Calculation of Powers - - -	52
XVIII. Of Roots with relation to Powers in general -	54
XIX. Of the Method of representing Irrational Numbers by Fractional Exponents - -	56
XX. Of the different Methods of Calculation, and of their Mutual Connexion - - -	60
XXI. Of Logarithms in general - - -	63
XXII. Of the Logarithmic Tables that are now in use -	66
XXIII. Of the Method of expressing Logarithms -	69

SECTION II.

Of the different Methods of calculating Compound Quantities.

Chap. I. Of the Addition of Compound Quantities -	76
II. Of the Subtraction of Compound Quantities -	78
III. Of the Multiplication of Compound Quantities -	79
IV. Of the Division of Compound Quantities -	84
V. Of the Resolution of Fractions into Infinite Series	88
VI. Of the Squares of Compound Quantities -	97

	Page
Chap. VII. Of the Extraction of Roots applied to Compound Quantities	- 100
VIII. Of the Calculation of Irrational Quantities	- 104
IX. Of Cubes, and of the Extraction of Cube Roots	107
X. Of the higher Powers of Compound Quantities	110
XI. Of the Transposition of the Letters, on which the demonstration of the preceding Rule is founded	- 115
XII. Of the Expression of Irrational Powers by Infinite Series	- 120
XIII. Of the Resolution of Negative Powers	- 123

SECTION III.

Of Ratios and Proportions.

Chap. I. Of Arithmetical Ratio, or the Difference between two Numbers	- 126
II. Of Arithmetical Proportion	- 129
III. Of Arithmetical Progressions	- 131
IV. Of the Summation of Arithmetical Progressions	135
V. Of Figurate, or Polygonal Numbers	- 139
VI. Of Geometrical Ratio	- 146
VII. Of the greatest Common Divisor of two given Numbers	- 148
VIII. Of Geometrical Proportions	- 152
IX. Observations on the Rules of Proportion and their Utility	- 155
X. Of Compound Relations	- 159
XI. Of Geometrical Progressions	- 164
XII. Of Infinite Decimal Fractions	- 171
XIII. Of the Calculation of Interest	- 177

SECTION IV.

Of Algebraic Equations, and of the Resolution of those Equations.

Chap. I. Of the Solution of Problems in General	- 186
II. Of the Resolution of Simple Equations, or Equations of the First Degree	- 189
III. Of the Solution of Questions relating to the preceding Chapter	- 194
IV. Of the Resolution of two or more Equations of the First Degree	- 206
V. Of the Resolution of Pure Quadratic Equations	216
VI. Of the Resolution of Mixed Equations of the Second Degree	- 222
VII. Of the Extraction of the Roots of Polygonal Numbers	- 230
VIII. Of the Extraction of Square Roots of Binomials	- 234

	Page
Chap. IX. Of the Nature of Equations of the Second Degree	- 241
X. Of Pure Equations of the Third Degree	- 248
XI. Of the Resolution of Complete Equations of the Third Degree	- 253
XII. Of the Rule of <i>Cardan</i> , or that of <i>Scipio Ferro</i>	262
XIII. Of the Resolution of Equations of the Fourth Degree	- 272
XIV. Of the Rule of <i>Bombelli</i> , for reducing the Resolution of Equations of the Fourth Degree to that of Equations of the Third Degree	- 278
XV. Of a new Method of resolving Equations of the Fourth Degree	- 282
XVI. Of the Resolution of Equations by Approximation	- 289

PART II.

PART II., *Containing the Analysis of Indeterminate Quantities.*

Chap. I. Of the Resolution of Equations of the First Degree, which contain more than one unknown Quantity	- 299
II. Of the Rule which is called <i>Regula Cæci</i> , for determining, by means of two Equations, three or more Unknown Quantities	- 312
III. Of Compound Indeterminate Equations, in which one of the Unknown Quantities does not exceed the First Degree	- 317
IV. Of the Method of rendering Surd Quantities, of the form $\sqrt{\frac{a}{x} + ax + cx^2}$, Rational	- 322
V. Of the Cases in which the Formula $a + bx + cx^2$ can never become a Square	- 335
VI. Of the Cases in Integer Numbers, in which the Formula $ax^2 + b$ becomes a Square	- 342
VII. Of a particular Method, by which the Formula $an^2 + 1$ becomes a Square in Integers	- 351
VIII. Of the Method of rendering the Irrational Formula $\sqrt{a + bx + cx^2 + dx^3}$ Rational	- 361
IX. Of the Method of rendering rational the incommensurable Formula $\sqrt{x + bx + cx^2 + dx^3 + ex^4}$	368
X. Of the Method of rendering rational the irrational Formula $\sqrt[3]{a + bx + cx^2 + dx^3}$	- 379
XI. Of the Resolution of the Formula $ax^2 + bxy + cy^2$ into its Factors	- 387
XII. Of the Transformation of the Formula $ax^3 + cy^2$ into Squares and higher Powers	- 396
XIII. Of some Expressions of the Form $ax^4 + by^4$, which are not reducible to Squares	- 405

	Page
Chap. XIV. Solution of some Questions that belong to this Part of Algebra	413
XV. Solutions of some Questions in which Cubes are required	449
ADDITIONS BY M. DE LA GRANGE.	
Advertisement	463
Chap. I. Of Continued Fractions	465
II. Solution of some New and Curious Arithmetical Problems	495
III. Of the Resolution in Integer Numbers of Equations of the First Degree containing two Unknown Quantities	530
IV. General Method for resolving in Integer Equations of two Unknown Quantities, one of which does not exceed the First Degree	534
V. A direct and general Method for finding the values of x , that will render Rational Quantities of the form $\sqrt{a + bx + cx^2}$, and for resolving, in Rational Numbers, the indeterminate Equations of the second Degree, which have two Unknown Quantities, when they admit of Solutions of this kind	537
Resolution of the Equation $Ap^2 + Bq^2 = z^2$ in Integer Numbers	539
VI. Of Double and Triple Equalities	547
VII. A direct and general Method for finding all the values of y expressed in Integer Numbers, by which we may render Quantities of the form $\sqrt{Ay^2 + B}$, rational; A and B being given Integer Numbers; and also for finding all the possible Solutions, in Integer Numbers, of indeterminate Quadratic Equations of two unknown Quantities.	550
Resolution of the Equation $cy^2 - 2nyz + Bz^2 = 1$ in Integer Numbers	552
First Method	ib.
Second Method	555
Of the Manner of finding all the possible Solutions of the Equations $cy^2 - 2nyz + Bz^2 = 1$, when we know only one of them	559
Of the Manner of finding all the possible Solutions, in whole Numbers, of Indeterminate Quadratic Equations of two Unknown Quantities	565
VIII. Remarks on Equations of the Form $p^2 = Aq^2 + 1$, and on the common Method of resolving them in whole Numbers	578
IX. Of the Manner of finding Algebraic Functions of all Degrees, which, when multiplied together, may always produce similar Functions	583