

19. If  $\sqrt{x} + \sqrt{a+x} = \frac{2a}{\sqrt{a+x}}$ , then will  $x = \frac{a}{3}$ .

20. If  $\sqrt{aa+xx} = \sqrt[4]{(b^4+x^4)}$ , then will  $x = \frac{\sqrt{(b^4-a^4)}}{2a^2}$ .

21. If  $y = \sqrt{a^2 + \sqrt{b^2 + x^2}} - a$ , then will  $x = \frac{bb}{4a} - a$ .

22. If  $\frac{128}{3x-4} = \frac{216}{5x-6}$ , then will  $x = 12$ .

23. If  $\frac{42x}{x-2} = \frac{35x}{x-3}$ , then will  $x = 8$ .

24. If  $\frac{45}{2x+3} = \frac{57}{4x-5}$ , then will  $x = 6$ .

25. If  $\frac{x^2-12}{3} = \frac{x^2-4}{4}$ , then will  $x = 6$ .

26. If  $615x - 7x^3 = 48x$ , then will  $x = 9$ .

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### CHAP. III.

*Of the Solution of Questions relating to the preceding Chapter.*

585. *Question 1.* To divide 7 into two such parts that the greater may exceed the less by 3.

Let the greater part be  $x$ , then the less will be  $7 - x$ ; so that  $x = 7 - x + 3$ , or  $x = 10 - x$ . Adding  $x$ , we have  $2x = 10$ ; and dividing by 2,  $x = 5$ .

The two parts therefore are 5 and 2.

*Question 2.* It is required to divide  $a$  into two parts, so that the greater may exceed the less by  $b$ .

Let the greater part be  $x$ , then the other will be  $a - x$ ; so that  $x = a - x + b$ . Adding  $x$ , we have  $2x = a + b$ ;

and dividing by 2,  $x = \frac{a+b}{2}$ .

Another method of solution. Let the greater part =  $x$ ; which as it exceeds the less by  $b$ , it is evident that this is less than the other by  $b$ , and therefore must be =  $x - b$ . Now,

these two parts, taken together, ought to make  $a$ ; so that  $2x - b = a$ ; adding  $b$ , we have  $2x = a + b$ , wherefore  $x = \frac{a+b}{2}$ , which is the value of the greater part; and that

of the less will be  $\frac{a+b}{2} - b$ , or  $\frac{a+b}{2} - \frac{2b}{2}$ , or  $\frac{a-b}{2}$ .

586. *Question 3.* A father leaves 1600 pounds to be divided among his three sons in the following manner; viz. the eldest is to have 200 pounds more than the second, and the second 100 pounds more than the youngest. Required the share of each.

Let the share of the third son be  $x$

Then the second's will be - -  $x + 100$ ; and

The first son's share - - -  $x + 300$ .

Now, these three sums make up together 1600*l.*; we have, therefore,

$$3x + 400 = 1600$$

$$3x = 1200$$

$$\text{and } x = 400$$

The share of the youngest is 400*l.*

That of the second is - - 500*l.*

That of the eldest is - - 700*l.*

587. *Question 4.* A father leaves to his four sons 8600*l.* and, according to the will, the share of the eldest is to be double that of the second, minus 100*l.*; the second is to receive three times as much as the third, minus 200*l.*; and the third is to receive four times as much as the fourth, minus 300*l.* What are the respective portions of these four sons?

Call the youngest son's share  $x$

Then the third son's is -  $4x - 300$

The second son's is - -  $12x - 1100$

And the eldest's - - -  $24x - 2300$

Now, the sum of these four shares must make 8600*l.* We have, therefore,  $41x - 3700 = 8600$ , or

$$41x = 12300, \text{ and } x = 300.$$

Therefore the youngest's share is 300*l.*

The third son's - - - - - 900*l.*

The second's - - - - - 2500*l.*

The eldest's - - - - - 4900*l.*

588. *Question 5.* A man leaves 11000 crowns to be divided between his widow, two sons, and three daughters. He intends that the mother should receive twice the share of a son, and that each son should receive twice as much

as a daughter. Required how much each of them is to receive.

Suppose the share of each daughter to be  $x$   
 Then each son's is consequently - - -  $2x$   
 And the widow's - - - - -  $4x$

The whole inheritance, therefore, is  $3x + 4x + 4x$ ; or  $11x = 11000$ , and, consequently,  $x = 1000$ .

Each daughter, therefore, is to receive 1000 crowns;  
 So that the three receive in all 3000  
 Each son receives 2000;  
 So that the two sons receive - 4000  
 And the mother receives - - 4000

Sum 11000 crowns

589. *Question 6.* A father intends by his will, that his three sons should share his property in the following manner: the eldest is to receive 1000 crowns less than half the whole fortune; the second is to receive 800 crowns less than the third of the whole; and the third is to have 600 crowns less than the fourth of the whole. Required the sum of the whole fortune, and the portion of each son.

Let the fortune be expressed by  $x$ :

The share of the first son is  $\frac{1}{2}x - 1000$   
 That of the second - - -  $\frac{1}{3}x - 800$   
 That of the third - - -  $\frac{1}{4}x - 600$

So that the three sons receive in all  $\frac{1}{2}x + \frac{1}{3}x + \frac{1}{4}x - 2400$ , and this sum must be equal to  $x$ . We have, therefore, the equation  $\frac{1}{12}x - 2400 = x$ ; and subtracting  $x$ , there remains  $\frac{1}{12}x - 2400 = 0$ ; then adding 2400, we have  $\frac{1}{12}x = 2400$ ; and, lastly, multiplying by 12, we obtain  $x = 28800$ .

The fortune, therefore, consists of 28800 crowns; and  
 The eldest son receives 13400 crowns  
 The second - - - - 8800  
 The youngest - - - 6600

28800 crowns.

590. *Question 7.* A father leaves four sons, who share his property in the following manner: the first takes the half of the fortune, minus 3000*l.*; the second takes the third, minus 1000*l.*; the third takes exactly the fourth of the property; and the fourth takes 600*l.* and the fifth part of the property. What was the whole fortune, and how much did each son receive?

Let the whole fortune be represented by  $x$  :

- Then the eldest son will have  $\frac{1}{2}x - 3000$
- The second - - - - -  $\frac{1}{3}x - 1000$
- The third - - - - -  $\frac{1}{4}x$
- The youngest - - - - -  $\frac{1}{5}x + 600$ .

And the four will have received in all  $\frac{1}{2}x + \frac{1}{3}x + \frac{1}{4}x + \frac{1}{5}x - 3400$ , which must be equal to  $x$ .

Whence results the equation  $\frac{77}{60}x - 3400 = x$ ; then subtracting  $x$ , we have  $\frac{17}{60}x - 3400 = 0$ ; adding 3400, we obtain  $\frac{17}{60}x = 3400$ ; then dividing by 17, we have  $\frac{1}{60}x = 200$ ; and multiplying by 60, gives  $x = 12000$ .

The fortune therefore consisted of 12000*l*.

- The first son received 3000
- The second - - - 3000
- The third - - - 3000
- The fourth - - - 3000

591. *Question 8.* To find a number such, that if we add to it its half, the sum exceeds 60 by as much as the number itself is less than 65.

Let the number be represented by  $x$  :

Then  $x + \frac{1}{2}x - 60 = 65 - x$ , or  $\frac{3}{2}x - 60 = 65 - x$ . Now, by adding  $x$ , we have  $\frac{5}{2}x - 60 = 65$ ; adding 60, we have  $\frac{5}{2}x = 125$ ; dividing by 5, gives  $\frac{1}{2}x = 25$ ; and multiplying by 2, we have  $x = 50$ .

Consequently, the number sought is 50.

592. *Question 9.* To divide 32 into two such parts, that if the less be divided by 6, and the greater by 5, the two quotients taken together may make 6.

Let the less of the two parts sought be  $x$ ; then the greater will be  $32 - x$ . The first, divided by 6, gives

$\frac{x}{6}$ ; and the second, divided by 5, gives  $\frac{32-x}{5}$ . Now  $\frac{x}{6} +$

$\frac{32-x}{5} = 6$ : so that multiplying by 5, we have  $\frac{5}{6}x + 32 -$

$x = 30$ , or  $-\frac{1}{6}x + 32 = 30$ ; adding  $\frac{1}{6}x$ , we have  $32 = 30 + \frac{1}{6}x$ ; subtracting 30, there remains  $2 = \frac{1}{6}x$ ; and lastly, multiplying by 6, we have  $x = 12$ .

So that the less part is 12, and the greater part is 20.

593. *Question 10.* To find such a number, that if multiplied by 5, the product shall be as much less than 40 as the number itself is less than 12.

Let the number be  $x$ ; which is less than 12 by  $12 - x$ ; then taking the number  $x$  five times, we have  $5x$ , which is

less than 40 by  $40 - 5x$ , and this quantity must be equal to  $12 - x$ .

We have, therefore,  $40 - 5x = 12 - x$ . Adding  $5x$ , we have  $40 = 12 + 4x$ ; and subtracting 12, we obtain  $28 = 4x$ ; lastly, dividing by 4, we have  $x = 7$ , the number sought.

594. *Question 11.* To divide 25 into two such parts, that the greater may be equal to 49 times the less.

Let the less part be  $x$ , than the greater will be  $25 - x$ ; and the latter divided by the former ought to give the

quotient 49: we have therefore  $\frac{25-x}{x} = 49$ . Multiplying

by  $x$ , we have  $25 - x = 49x$ ; adding  $x$ , we have  $25 = 50x$ ; and dividing by 50, gives  $x = \frac{1}{2}$ .

The less of the two numbers is  $\frac{1}{2}$ , and the greater is  $24\frac{1}{2}$ ; dividing therefore the latter by  $\frac{1}{2}$ , or multiplying by 2, we obtain 49.

595. *Question 12.* To divide 48 into nine parts, so that every part may be always  $\frac{1}{2}$  greater than the part which precedes it.

Let the first, or least part be  $x$ , then the second will be  $x + \frac{1}{2}$ , the third  $x + 1$ , &c.

Now, these parts form an arithmetical progression, whose first term is  $x$ ; therefore the ninth and last term will be  $x + 4$ . Adding those two terms together, we have  $2x + 4$ ; multiplying this quantity by the number of terms, or by 9, we have  $18x + 36$ ; and dividing this product by 2, we obtain the sum of all the nine parts =  $9x + 18$ ; which ought to be equal to 48. We have, therefore,  $9x + 18 = 48$ ; subtracting 18, there remains  $9x = 30$ ; and dividing by 9, we have  $x = 3\frac{1}{3}$ .

The first part, therefore, is  $3\frac{1}{3}$ , and the nine parts will succeed in the following order:

$$1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 7 \quad 8 \quad 9 \\ 3\frac{1}{3} + 3\frac{2}{3} + 4\frac{1}{3} + 4\frac{2}{3} + 5\frac{1}{3} + 5\frac{2}{3} + 6\frac{1}{3} + 6\frac{2}{3} + 7\frac{1}{3}.$$

Which together make 48.

596. *Question 13.* To find an arithmetical progression, whose first term is 5, the last term 10, and the entire sum 60.

Here we know neither the difference nor the number of terms; but we know that the first and the last term would enable us to express the sum of the progression, provided only the number of terms were given. We shall therefore suppose this number to be  $x$ , and express the sum of the

progression by  $\frac{15x}{2}$ . We know also, that this sum is 60;

so that  $\frac{15x}{2} = 60$ ; or  $\frac{1}{2}x = 4$ , and  $x = 8$ .

Now, since the number of terms is 8, if we suppose the difference to be  $z$ , we have only to seek for the eighth term upon this supposition, and to make it equal to 10. The second term is  $5 + z$ , the third is  $5 + 2z$ , and the eighth is  $5 + 7z$ ; so that

$$\begin{aligned} 5 + 7z &= 10 \\ 7z &= 5 \\ \text{and } z &= \frac{5}{7}. \end{aligned}$$

The difference of the progression, therefore, is  $\frac{5}{7}$ , and the number of terms is 8; consequently, the progression is

$$\begin{array}{cccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 5 + 5\frac{5}{7} & + & 6\frac{3}{7} & + & 7\frac{1}{7} & + & 7\frac{6}{7} & + & 8\frac{4}{7} & + & 9\frac{2}{7} & + & 10, \end{array}$$

the sum of which is 60.

597. *Question 14.* To find such a number, that if 1 be subtracted from its double, and the remainder be doubled, from which if 2 be subtracted, and the remainder divided by 4, the number resulting from these operations shall be 1 less than the number sought.

Suppose this number to be  $x$ ; the double is  $2x$ ; subtracting 1, there remains  $2x - 1$ ; doubling this, we have  $4x - 2$ ; subtracting 2, there remains  $4x - 4$ ; dividing by 4, we have  $x - 1$ ; and this must be 1 less than  $x$ ; so that

$$x - 1 = x - 1.$$

But this is what is called an *identical equation*; and shews that  $x$  is indeterminate; or that any number whatever may be substituted for it.

598. *Question 15.* I bought some ells of cloth at the rate of 7 crowns for 5 ells, which I sold again at the rate of 11 crowns for 7 ells, and I gained 100 crowns by the transaction. How much cloth was there?

Supposing the number of ells to be  $x$ , we must first see how much the cloth cost; which is found by the following proportion:

$$\text{As } 5 : x :: 7 : \frac{7x}{5} \text{ the price of the ells.}$$

This being the expenditure; let us now see the receipt: in order to which, we must make the following proportion:

E. C. E.

As  $7 : 11 :: x : \frac{11}{7}x$  crowns;

and this receipt ought to exceed the expenditure by 100 crowns. We have, therefore, this equation:

$$\frac{11}{7}x = \frac{7}{5}x + 100.$$

Subtracting  $\frac{7}{5}x$ , there remains  $\frac{6}{35}x = 100$ ; therefore  $6x = 3500$ , and  $x = 583\frac{1}{3}$ .

There were, therefore,  $583\frac{1}{3}$  ells bought for  $816\frac{2}{3}$  crowns, and sold again for  $916\frac{2}{3}$  crowns; by which means the profit was 100 crowns.

599. *Question 16.* A person buys 12 pieces of cloth for 140*l.*; of which two are white, three are black, and seven are blue: also, a piece of the black cloth costs two pounds more than a piece of the white, and a piece of the blue cloth costs three pounds more than a piece of the black. Required the price of each kind.

Let the price of a white piece be  $x$  pounds; then the two pieces of this kind will cost  $2x$ ; also, a black piece costing  $x + 2$ , the three pieces of this color will cost  $3x + 6$ ; and lastly, as a blue piece costs  $x + 5$ , the seven blue pieces will cost  $7x + 35$ : so that the twelve pieces amount in all to  $12x + 41$ .

Now, the known price of these twelve pieces is 140 pounds; we have, therefore,  $12x + 41 = 140$ , and  $12x = 99$ ; wherefore  $x = 8\frac{1}{4}$ . So that

A piece of white cloth costs  $8\frac{1}{4}$ *l.*

A piece of black cloth costs  $10\frac{1}{4}$ *l.*

A piece of blue cloth costs  $13\frac{1}{4}$ *l.*

600. *Question 17.* A man having bought some nutmegs, says that three of them cost as much more than one penny, as four cost him more than two pence halfpenny. Required the price of the nutmegs?

Let  $x$  be the excess of the price of three nuts above one penny, or four farthings. Now, if three nutmegs cost  $x + 4$  farthings, four will cost, by the condition of the question,  $x + 10$  farthings; but the price of three nutmegs gives that of four in another way, namely, by the Rule of Three. Thus,

$$3 : x + 4 :: 4 : \frac{4x + 16}{3}.$$

So that  $\frac{4x + 16}{3} = x + 10$ ; or,  $4x + 16 = 3x + 30$ ;

therefore  $x + 16 = 30$ , and  $x = 14$ .

Three nutmegs, therefore, cost  $4\frac{1}{2}d.$ , and four cost  $6d.$  : wherefore each costs  $1\frac{1}{2}d.$

601. *Question 18.* A certain person has two silver cups, and only one cover for both. The first cup weighs 12 ounces; and if the cover be put on it, it weighs twice as much as the other cup: but when the other cup has the cover, it weighs three times as much as the first. Required the weight of the second cup, and that of the cover.

Suppose the weight of the cover to be  $x$  ounces; then the first cup being covered it will weigh  $x + 12$ ; and this weight being double that of the second, the second cup must weigh  $\frac{1}{2}x + 6$ ; and, with the cover, it will weigh  $x + \frac{1}{2}x + 6$ ,  $\frac{3}{2}x + 6$ ; which weight ought to be the triple of 12; that is, three times the weight of the first cup. We shall therefore have the equation  $\frac{3}{2}x + 6 = 36$ , or  $\frac{3}{2}x = 30$ ; so that  $\frac{1}{2}x = 10$  and  $x = 20$ .

The cover, therefore, weighs 20 ounces, and the second cup weighs 16 ounces.

602. *Question 19.* A banker has two kinds of change: there must be  $a$  pieces of the first to make a crown; and  $b$  pieces of the second to make the same. Now, a person wishes to have  $c$  pieces for a crown. How many pieces of each kind must the banker give him?

Suppose the banker gives  $x$  pieces of the first kind; it is evident that he will give  $c - x$  pieces of the other kind;

but the  $x$  pieces of the first are worth  $\frac{x}{a}$  crown, by the pro-

portion  $a : x :: 1 : \frac{x}{a}$ ; and the  $c - x$  pieces of the second

kind are worth  $\frac{c-x}{b}$  crown, because we have  $b : c - x :: 1 :$

$\frac{c-x}{b}$ . So that,  $\frac{x}{a} + \frac{c-x}{b} = 1$ ;

$$\text{or } \frac{bx}{a} + c - x = b; \text{ or } bx + ac - ax = ab;$$

$$\text{or, rather } bx - ax = ab - ac;$$

$$\text{whence we have } x = \frac{ab - ac}{b - a}, \text{ or } x = \frac{a(b - c)}{b - a};$$

consequently,  $c - x$ , the pieces of the second kind, must be

$$= \frac{bc - ab}{b - a} = \frac{b(c - a)}{b - a}.$$



The banker must therefore give  $\frac{a(b-c)}{b-a}$  pieces of the first kind, and  $\frac{b(c-a)}{b-a}$  pieces of the second kind.

*Remark.* These two numbers are easily found by the Rule of Three, when it is required to apply the results which we have obtained. Thus, to find the first we say,

$b - a : a :: b - c : \frac{a(b-c)}{b-a}$ ; and the second number is

found thus;  $b - a : b :: c - a : \frac{b(c-a)}{b-a}$ .

It ought to be observed also, that  $a$  is less than  $b$ , and that  $c$  is less than  $b$ ; but at the same time greater than  $a$ , as the nature of the thing requires.

603. *Question 20.* A banker has two kinds of change; 10 pieces of one make a crown, and 20 pieces of the other make a crown; and a person wishes to change a crown into 17 pieces of money: how many of each sort must he have?

We have here  $a = 10$ ,  $b = 20$ , and  $c = 17$ , which furnishes the following proportions:

First,  $10 : 10 :: 3 : 3$ , so that the number of pieces of the first kind is 3.

Secondly,  $10 : 20 :: 7 : 14$ , and the number of the second kind is 14.

604. *Question 21.* A father leaves at his death several children, who share his property in the following manner: namely, the first receives a hundred pounds, and the tenth part of the remainder; the second receives two hundred pounds, and the tenth part of the remainder; the third takes three hundred pounds, and the tenth part of what remains; and the fourth takes four hundred pounds, and the tenth part of what then remains; and so on. And it is found that the property has thus been divided equally among all the children. Required how much it was, how many children there were, and how much each received?

This question is rather of a singular nature, and therefore deserves particular attention. In order to resolve it more easily, we shall suppose the whole fortune to be  $z$  pounds; and since all the children receive the same sum, let the share of each be  $x$ , by which means the number of children will be expressed by  $\frac{z}{x}$ . Now, this being laid down, we may proceed

to the solution of the question, as follows:

Sum, or property to be divided.	Order of the children.	Portion of each.	Differences.
$z$	1st	$x = 100 + \frac{z-100}{10}$	
$z-x$	2d	$x = 200 + \frac{z-x-200}{10}$	$100 - \frac{x-100}{10} = 0$
$z-2x$	3d	$x = 300 + \frac{z-2x-300}{10}$	$100 - \frac{x-100}{10} = 0$
$z-3x$	4th	$x = 400 + \frac{z-3x-400}{10}$	$100 - \frac{x-100}{10} = 0$
$z-4x$	5th	$x = 500 + \frac{z-4x-500}{10}$	$100 - \frac{x-100}{10} = 0$
$z-5x$	6th	$x = 600 + \frac{z-5x-600}{10}$	and so on.

We have inserted, in the last column, the differences which we obtain by subtracting each portion from that which follows; but all the portions being equal, each of the differences must be = 0. As it happens also, that all these differences are expressed exactly alike, it will be sufficient to make one of them equal to nothing, and we shall have the equation  $100 - \frac{x-100}{10} = 0$ . Here, multiplying by 10, we

have  $1000 - x - 100 = 0$ , or  $900 - x = 0$ ; and, consequently,  $x = 900$ .

We know now, therefore, that the share of each child was 900; so that taking any one of the equations of the third column, the first, for example, it becomes, by substituting the value of  $x$ ,  $900 = 100 + \frac{z-100}{10}$ , whence we immediately obtain the value of  $z$ ; for we have

$$9000 = 1000 + z - 100, \text{ or } 9000 = 900 + z;$$

therefore  $z = 8100$ ; and consequently  $\frac{z}{x} = 9$ .

So that the number of children was 9; the fortune left by the father was 8100 pounds; and the share of each child was 900 pounds.

QUESTIONS FOR PRACTICE.

1. To find a number, to which if there be added a half, a third, and a fourth of itself, the sum will be 50. *Ans.* 24.

2. A person being asked what his age was, replied that  $\frac{3}{4}$  of his age multiplied by  $\frac{1}{12}$  of his age gives a product equal to his age. What was his age? *Ans.* 16.

3. The sum of 660*l.* was raised for a particular purpose by four persons, A, B, C, and D; B advanced twice as much as A; C as much as A and B together; and D as much as B and C. What did each contribute?

*Ans.* 60*l.*, 120*l.*, 180*l.*, and 300*l.*

4. To find that number whose  $\frac{1}{3}$  part exceeds its  $\frac{1}{4}$  part by 12. *Ans.* 144.

5. What sum of money is that whose  $\frac{1}{3}$  part,  $\frac{1}{4}$  part, and  $\frac{1}{5}$  part, added together, shall amount to 94 pounds?

*Ans.* 120*l.*

6. In a mixture of copper, tin, and lead, one half of the whole — 16*lb.* was copper; one-third of the whole — 12*lb.* tin; and one-fourth of the whole — 4*lb.* lead: what quantity of each was there in the composition?

*Ans.* 128*lb.* of copper, 84*lb.* of tin, and 76*lb.* of lead.

7. A bill of 120*l.* was paid in guineas and moidores, and the number of pieces of both sorts were just 100; to find how many there were of each. *Ans.* 50.

8. To find two numbers in the proportion of 2 to 1, so that if 4 be added to each, the two sums shall be in the proportion of 3 to 2. *Ans.* 4 and 8.

9. A trader allows 100*l.* per annum for the expenses of his family, and yearly augments that part of his stock which is not so expended, by a third part of it; at the end of three years, his original stock was doubled: what had he at first?

*Ans.* 1480*l.*

10. A fish was caught whose tail weighed 9*lb.* His head weighed as much as his tail and  $\frac{1}{2}$  his body; and his body weighed as much as his head and tail: what did the whole fish weigh?

*Ans.* 72*lb.*

11. One has a lease for 99 years; and being asked how much of it was already expired, answered, that two-thirds of the time past was equal to four-fifths of the time to come: required the time past. *Ans.* 54 years.

12. It is required to divide the number 48 into two such parts, that the one part may be three times as much above 20, as the other wants of 20. *Ans.* 32 and 16.

13. One rents 25 acres of land at 7 pounds 12 shillings per annum; this land consisting of two sorts, he rents the better sort at 8 shillings per acre, and the worse at 5: required the number of acres of the better sort.

*Ans.* 9 of the better.

14. A certain cistern, which would be filled in 12 minutes

by two pipes running into it, would be filled in 20 minutes by one alone. Required in what time it would be filled by the other alone. *Ans.* 30 minutes.

15. Required two numbers, whose sum may be  $s$ , and their proportion as  $a$  to  $b$ . *Ans.*  $\frac{as}{a+b}$  and  $\frac{bs}{a+b}$ .

16. A privateer, running at the rate of 10 miles an hour, discovers a ship 18 miles off making way at the rate of 8 miles an hour: it is demanded how many miles the ship can run before she will be overtaken? *Ans.* 72.

17. A gentleman distributing money among some poor people, found that he wanted 10s. to be able to give 5s. to each; therefore he gives 4s. only, and finds that he has 5s. left: required the number of shillings and of poor people. *Ans.* 15 poor, and 65 shillings.

18. There are two numbers whose sum is the 6th part of their product, and the greater is to the less as 3 to 2. Required those numbers. *Ans.* 15 and 10.

*N. B.* This question may be solved by means of one unknown letter.

19. To find three numbers, so that the first, with half the other two, the second with one-third of the other two, and the third with one fourth of the other two, may be equal to 34. *Ans.* 26, 22, and 10.

20. To find a number consisting of three places, whose digits are in arithmetical progression: if this number be divided by the sum of its digits, the quotients will be 48; and if from the number 198 be subtracted, the digits will be inverted. *Ans.* 432.

21. To find three numbers, so that  $\frac{1}{2}$  the first,  $\frac{1}{3}$  of the second, and  $\frac{1}{4}$  of the third, shall be equal to 62:  $\frac{1}{3}$  of the first,  $\frac{1}{4}$  of the second, and  $\frac{1}{5}$  of the third, equal to 47; and  $\frac{1}{4}$  of the first,  $\frac{1}{5}$  of the second, and  $\frac{1}{6}$  of the third, equal to 38. *Ans.* 24, 60, 120.

22. If A and B, together, can perform a piece of work in 8 days; A and C together in 9 days; and B and C in 10 days; how many days will it take each person, alone, to perform the same work? *Ans.*  $14\frac{3}{4}$ ,  $17\frac{2}{3}$ ,  $23\frac{7}{11}$ .

23. What is that fraction which will become equal to  $\frac{1}{3}$ , if an unit be added to the numerator; but on the contrary, if an unit be added to the denominator, it will be equal to  $\frac{1}{4}$ ? *Ans.*  $\frac{1}{15}$ .

24. The dimensions of a certain rectangular floor are such, that if it had been 2 feet broader, and 3 feet longer, it would have been 64 square feet larger; but if it had been 3

feet broader and 2 feet longer, it would then have been 68 square feet larger: required the length and breadth of the floor.

*Ans.* Length 14 feet, and breadth 10 feet.

25. A hare is 50 leaps before a greyhound, and takes 4 leaps to the greyhound's 3; but two of the greyhound's leaps are as much as three of the hare's: how many leaps must the greyhound take to catch the hare? *Ans.* 300.

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## CHAP. IV.

### *Of the Resolution of two or more Equations of the First Degree.*

605. It frequently happens that we are obliged to introduce into algebraic calculations two or more unknown quantities, represented by the letters  $x, y, z$ : and if the question is determinate, we are brought to the same number of equations as there are unknown quantities; from which it is then required to deduce those quantities. As we consider, at present, those equations only, which contain no powers of an unknown quantity higher than the first, and no products of two or more unknown quantities, it is evident that all those equations have the form

$$ax + by + cx = d.$$

606. Beginning therefore with two equations, we shall endeavour to find from them the value of  $x$  and  $y$ : and, in order that we may consider this case in a general manner, let the two equations be,

$$ax + by = c; \text{ and } fx + gy = h;$$

in which,  $a, b, c$ , and  $f, g, h$ , are known numbers. It is required, therefore, to obtain, from these two equations, the two unknown quantities  $x$  and  $y$ .

607. The most natural method of proceeding will readily present itself to the mind; which is, to determine, from both equations, the value of one of the unknown quantities, as for example  $x$ , and to consider the equality of those two values; for then we shall have an equation, in which the unknown quantity  $y$  will be found by itself, and may be determined by the rules already given. Then, knowing  $y$ , we shall have only to substitute its value in one of the quantities that express  $x$ .