475. Let the two proportions be 6 : 4 : : 15 : 10 and 9 : 12 : : 15 : 20, their combination will give the proportion $6 \times 9 : 4 \times 12 :: 15 \times 15 : 10 \times 20$,

or 54 : 48 : : 225 : 200,
or 9 : 8 : : 9 : 8.

476. We shall observe, lastly, that if two products are equal, $ad = bc$, we may reciprocally convert this equality into a geometrical proportion; for we shall always have one of the factors of the first product in the same proportion to one of the factors of the second product, as the other factor of the second product is to the other factor of the first product: that is, in the present case, $a : c :: b : d$, or $a : b :: c : d$. Let $3 \times 8 = 4 \times 6$, and we may form from it this proportion, 8 : 4 : : 6 : 3, or this, 3 : 4 : : 6 : 8. Likewise, if $3 \times 5 = 1 \times 15$, we shall have 3 : 15 : : 1 : 5, or 5 . 1 : : 15 : 3, or 3 : 1 : : 15 : 5.

CHAP. IX.

Observations on the Rules *of* Proportion *and their* Utility.

477. This theory is so useful in the common occurrences of life, that scarcely any person can do without it. There is always a proportion between prices and commodities; and when different kinds of money are the subject of exchange, the whole consists in determining their mutual relations. The examples furnished by these reflections will be very proper for illustrating the principles of proportion, and shewing their utility by the application of them.

478. If we wished to know, for example, the relation between two kinds of money; suppose an old *louis d'or* and a *ducat:* we must first know the value of those pieces when compared with others of the same kind. Thus, an old louis being, at Berlin, worth 5 rixdollars and 8 drachms, and a ducat being worth 3 rixdollars, we may reduce these two values to one denomination; either to rixdollars, which gives the proportion 1L : 1D : : $5\frac{1}{3}$R : 3R, or : : 16 : 9; or to drachms, in which case we have 1L : 1D : : 128 : 72 : : 16 : 9; which proportions evidently give the true relation of the old louis to the ducat; for the equality of the products of the extremes and the means gives, in both cases, 9 louis

= 16 ducats; and, by means of this comparison, we may change any sum of old louis into ducats, and vice-versa. Thus, suppose it were required to find how many ducats there are in 1000 old louis, we have this proportion:

Lou. Lou. Duc. Duc.
As 9 : 1000 : : 16 : $1777\frac{7}{9}$, the number sought.

If, on the contrary, it were required to find how many old louis d'or there are in 1000 ducats, we have the following proportion:

Duc. Duc. Lou.
As 16 : 1000 : : 9 : $562\frac{1}{2}$ louis. *Ans.*

479. At Petersburgh the value of the ducat varies, and depends on the course of exchange; which course determines the value of the ruble in stivers, or Dutch halfpence, 105 of which make a ducat. So that when the exchange is at 45 stivers per ruble, we have this proportion:

As 45 : 105 : : 3 : 7;

and hence this equality, 7 rubles = 3 ducats.

Hence again we shall find the value of a ducat in rubles; for

Du. Du. Ru.
As 3 : 1 : : 7 : $2\frac{1}{3}$ rubles;

that is, 1 ducat is equal to $2\frac{1}{3}$ rubles.

But if the exchange were at 50 stivers, the proportion would be,

As 50 : 105 : : 10 : 21;

which would give 21 rubles = 10 ducats; whence 1 ducat = $2\frac{1}{10}$ rubles. Lastly, when the exchange is at 44 stivers, we have

As 44 : 105 : : 1 : $2\frac{17}{44}$ rubles;

which is equal to 2 rubles, $38\frac{7}{11}$ copecks.

480. It follows also from this, that we may compare different kinds of money, which we have frequently occasion to do in bills of exchange.

Suppose, for example, that a person of Petersburgh has 1000 rubles to be paid to him at Berlin, and that he wishes to know the value of this sum in ducats at Berlin.

The exchange is at $47\frac{1}{2}$; that is to say, one ruble makes $47\frac{1}{2}$ stivers; and in Holland, 20 stivers make a florin; $2\frac{1}{2}$ Dutch florins make a Dutch dollar: also, the exchange of Holland with Berlin is at 142; that is to say, for 100 Dutch dollars, 142 dollars are paid at Berlin; and lastly, the ducat is worth 3 dollars at Berlin.

481. To resolve the question proposed, let us proceed

step by step. Beginning therefore with the stivers, since 1 ruble = $47\frac{1}{2}$ stivers, or 2 rubles = 95 stivers, we shall have

Ru. Ru. Stiv.
As 2 : 1000 : : 95 : 47500 stivers;

then again,

Stiv. Stiv. Flor.
As 20 : 47500 : : 1 : 2375 florins.

Also, since $2\frac{1}{2}$ florins = 1 Dutch dollar, or 5 florins = 2 Dutch dollars; we shall have

Flor. Flor. D.D.
As 5 : 2375 : : 2 : 950 Dutch dollars.

Then, taking the dollars of Berlin, according to the exchange, at 142, we shall have

D.D. D.D. Dollars.
As 100 : 950 : : 142 : 1349 dollars of Berlin.

And lastly,

Dol. Dol. Du.
As 3 : 1349 : : 1 : $449\frac{2}{3}$ ducats,

which is the number sought.

482. Now, in order to render these calculations still more complete, let us suppose that the Berlin banker refuses, under some pretext or other, to pay this sum, and to accept the bill of exchange without five per cent. discount; that is, paying only 100 instead of 105. In that case, we must make use of the following proportion:

As 105 : 100 : : $449\frac{2}{3}$: $428\frac{16}{63}$ ducats;

which is the answer under those conditions.

483. We have shewn that six operations are necessary in making use of the Rule of Three; but we can greatly abridge those calculations by a rule which is called the *Rule of Reduction*, or *Double Rule of Three*. To explain which, we shall first consider the two antecedents of each of the six preceding operations:

1st. 2 rubles	:	95 stivers.
2d. 20 stivers	:	1 Dutch florin.
3d. 5 Dutch flor.	:	2 Dutch dollars.
4th. 100 Dutch doll.	:	142 dollars.
5th. 3 dollars	:	1 ducat.
6th. 105 ducats	:	100 ducats.

If we now look over the preceding calculations, we shall observe, that we have always multiplied the given sum by the third terms, or second antecedents, and divided the products by the first: it is evident, therefore, that we shall arrive at the same results by multiplying at once the sum proposed by the product of all the third terms, and dividing by the product of all the first terms: or, which amounts to the same thing, that we have only to make the following proportion: As the product of all the first terms, is to the given number of rubles, so is the product of all the second terms, to the number of ducats payable at Berlin.

484. This calculation is abridged still more, when amongst the first terms some are found that have common divisors with the second or third terms; for, in this case, we destroy those terms, and substitute the quotient arising from the division by that common divisor. The preceding example will, in this manner, assume the following form.

As (2.20.5.100.3 105) : 1000 : : (95.2.142.100) :

$\frac{1000.95.2.142.100}{2.20.5.100.3.105}$; and after cancelling the common divisors in the numerator and denominator, this will become

$\frac{10.19.142}{3.21} = \frac{26980}{63} = 428\frac{16}{63}$ ducats, as before.

485. The method which must be observed in using the Rule of Reduction is this: we begin with the kind of money in question, and compare it with another which is to begin the next relation, in which we compare this second kind with a third, and so on. Each relation, therefore, begins with the same kind as the preceding relation ended with; and the operation is continued till we arrive at the kind of money which the answer requires; at the end of which we must reckon the fractional remainders.

486. Let us give some other examples, in order to facilitate the practice of this calculation.

If ducats gain at Hamburgh 1 per cent. on two dollars banco; that is to say, if 50 ducats are worth, not 100, but 101 dollars banco; and if the exchange between Hamburgh and Konigsberg is 119 drachms of Poland; that is, if 1 dollar banco is equal to 119 Polish drachms; how many Polish florins are equivalent to 1000 ducats?

It being understood that 30 Polish drachms make 1 Polish florin,

Here	1 : 1000 : :	2 dollars banco	
	100	—	101 dollars banco
	1	—	119 Polish drachms
	30	—	1 Polish florin;

therefore,

$(100.30) : 1000 :: (2.101.119) : \frac{1000.2.101.119}{100.30} =$
$\frac{2.101.119}{3} = 8012\frac{2}{3}$ Polish florins. *Ans.*

487. We will propose another example, which may still farther illustrate this method.

Ducats of Amsterdam are brought to Leipsic, having in the former city the value of 5 flor. 4 stivers current; that is to say, 1 ducat is worth 104 stivers, and 5 ducats are worth 26 Dutch florins. If, therefore, the *agio of the bank* at Amsterdam is 5 per cent.; that is, if 105 currency are equal to 100 banco; and if the exchange from Leipsic to Amsterdam, in bank money, is $133\frac{1}{4}$ per cent.; that is, if for 100 dollars we pay at Leipsic $133\frac{1}{4}$ dollars; and lastly, 2 Dutch dollars making 5 Dutch florins; it is required to determine how many dollars we must pay at Leipsic, according to these exchanges, for 1000 ducats?

By the rule,

5 : 1000	: :	26 flor. Dutch curr.
105	—	100 flor. Dutch banco
400	—	533 doll. of Leipsick
5	—	2 doll. banco;

therefore,

As $(5.105.400.5) : 1000 :: (26.100.533.2) :$
$\frac{1000.26.100.533.2}{5.105.400.5} = \frac{4.26.533}{21} = 2639\frac{13}{21}$ dollars, the number sought.

CHAP. X.

Of Compound Relations.

488. *Compound Relations* are obtained by multiplying the terms of two or more relations, the antecedents by the antecedents, and the consequents by the consequents; we then say, that the relation between those two products is *compounded* of the relations given.

Thus the relations $a : b$, $c : d$, $e : f$, give the compound relation $ace : bdf$*.

* Each of these three *ratios* is said to be one of the *roots* of the compound ratio.