CHAP. VI.

Of Geometrical Ratio.

440. The *Geometrical ratio* of two numbers is found by resolving the question, *How many times* is one of those numbers greater than the other? This is done by dividing one by the other; and the quotient will express the ratio required.

441. We have here three things to consider; 1st, the first of the two given numbers, which is called the *antecedent;* 2dly, the other number, which is called the *consequent;* 3dly, the ratio of the two numbers, or the quotient arising from the division of the antecedent by the consequent. For example, if the relation of the numbers 18 and 12 be required, 18 is the antecedent, 12 is the consequent, and the ratio will be $\frac{18}{12} = 1\frac{1}{2}$; whence we see that the antecedent contains the consequent once and a half.

442. It is usual to represent geometrical relation by two points, placed one above the other, between the antecedent and the consequent. Thus, $a : b$ means the geometrical relation of these two numbers, or the ratio of a to b.

We have already remarked that this sign is employed to represent division *, and for this reason we make use of it here; because, in order to know the ratio, we must divide a by b; the relation expressed by this sign being read simply, a is to b.

443. Relation therefore is expressed by a fraction, whose numerator is the antecedent, and whose denominator is the consequent; but perspicuity requires that this fraction should be always reduced to its lowest terms: which is done, as we have already shewn, by dividing both the numerator and denominator by their greatest common divisor. Thus, the fraction $\frac{18}{12}$ becomes $\frac{3}{2}$, by dividing both terms by 6.

The algebraists of the sixteenth and seventeenth centuries paid great attention to these different kinds of numbers and their mutual connexion, and they discovered in them a variety of curious properties; but as their utility is not great, they are now seldom introduced into the systems of mathematics. F. T.

* It will be observed that we have made use of the symbol $\div$ for division, as is now usually done in books on this subject.

444. So that relations only differ according as their ratios are different; and there are as many different kinds of geometrical relations as we can conceive different ratios.

The first kind is undoubtedly that in which the ratio becomes unity. This case happens when the two numbers are equal, as in $3 : 3 :: 4 : 4 :: a : a$; the ratio is here 1, and for this reason we call it the relation of equality.

Next follow those relations in which the ratio is another whole number. Thus, $4 : 2$ the ratio is 2, and is called *double* ratio; $12 : 4$ the ratio is 3, and is called *triple* ratio: $24 : 6$ the ratio is 4, and is called *quadruple* ratio, &c.

We may next consider those relations whose ratios are expressed by fractions; such as $12 : 9$, where the ratio is $\frac{4}{3}$, or $1\frac{1}{3}$; and $18 : 27$, where the ratio is $\frac{2}{3}$, &c. We may also distinguish those relations in which the consequent contains exactly twice, thrice, &c. the antecedent: such are the relations $6 : 12$, $5 : 15$, &c. the ratio of which some call *subduple*, *subtriple*, &c. ratios.

Farther, we call that ratio *rational* which is an expressible number; the antecedent and consequent being integers, such as $11 : 7$, $8 : 15$, &c. and we call that an *irrational* or *surd* ratio, which can neither be exactly expressed by integers, nor by fractions, such as $\sqrt{5} : 8$, or $4 : \sqrt{3}$.

445. Let a be the antecedent, b the consequent, and d the ratio, we know already that a and b being given, we find $d = \frac{a}{b}$: if the consequent b were given with the ratio, we should find the antecedent $a = bd$, because bd divided by b gives d: and lastly, when the antecedent a is given, and the ratio d, we find the consequent $b = \frac{a}{d}$; for, dividing the antecedent a by the consequent $\frac{a}{d}$, we obtain the quotient d, that is to say, the ratio.

446. Every relation $a : b$ remains the same, if we multiply or divide the antecedent and consequent by the same number, because the ratio is the same: thus, for example, let d be the ratio of $a : b$, we have $d = \frac{a}{b}$; now the ratio of the relation $na : nb$ is also $\frac{na}{nb} = d$, and that of the relation $\frac{a}{n} : \frac{b}{n}$ is likewise $\frac{na}{nb} = d$.

447. When a ratio has been reduced to its lowest terms,

it is easy to perceive and enunciate the relation. For example, when the ratio $\frac{a}{b}$ has been reduced to the fraction $\frac{p}{q}$, we say $a : b = p : q$, or $a : b :: p : q$, which is read, a is to b as p is to q. Thus, the ratio of 6 : 3 being $\frac{2}{1}$, or 2, we say 6 : 3 : : 2 : 1. We have likewise 18 : 12 : : 3 : 2, and 24 : 18 : : 4 : 3, and 30 : 45 : : 2 : 3, &c. But if the ratio cannot be abridged, the relation will not become more evident; for we do not simplify it by saying 9 : 7 : : 9 . 7.

448. On the other hand, we may sometimes change the relation of two very great numbers into one that shall be more simple and evident, by reducing both to their lowest terms. Thus, for example, we can say, 28844 : 14422 : : 2 : 1; or, 10566 : 7044 : : 3 : 2; or, 57600 : 25200 : : 16 : 7.

449. In order, therefore, to express any relation in the clearest manner, it is necessary to reduce it to the smallest possible numbers; which is easily done, by dividing the two terms of it by their greatest common divisor. Thus, to reduce the relation 57600 : 25200 to that of 16 : 7, we have only to perform the single operation of dividing the numbers 57600 and 25200 by 3600, which is their greatest common divisor.

450. It is important, therefore, to know how to find the greatest common divisor of two given numbers; but this requires a Rule, which we shall explain in the following chapter.

CHAP. VII.

Of the Greatest Common Divisor *of two given* Numbers.

451. There are some numbers which have no other common divisor than unity; and when the numerator and denominator of a fraction are of this nature, it cannot be reduced to a more convenient form *. The two numbers 48 and 35, for example, have no common divisor, though each has its own divisors; for which reason, we cannot express the relation 48 : 35 more simply, because the division of two numbers by 1 does not diminish them.

* In this case, the two numbers are said to be prime to each other. See Art. 66.