$$(100.30) : 1000 :: (2.101.119) : \frac{1000.2.101.119}{100.30} =$$
$$\frac{2.101.119}{3} = 8012\tfrac{2}{3}$$ Polish florins. *Ans.*

487. We will propose another example, which may still farther illustrate this method.

Ducats of Amsterdam are brought to Leipsic, having in the former city the value of 5 flor. 4 stivers current; that is to say, 1 ducat is worth 104 stivers, and 5 ducats are worth 26 Dutch florins. If, therefore, the *agio of the bank* at Amsterdam is 5 per cent.; that is, if 105 currency are equal to 100 banco; and if the exchange from Leipsic to Amsterdam, in bank money, is $133\frac{1}{4}$ per cent.; that is, if for 100 dollars we pay at Leipsic $133\frac{1}{4}$ dollars; and lastly, 2 Dutch dollars making 5 Dutch florins; it is required to determine how many dollars we must pay at Leipsic, according to these exchanges, for 1000 ducats?

By the rule,

5 : 1000	: :	26 flor. Dutch curr.
105	—	100 flor. Dutch banco
400	—	533 doll. of Leipsick
5	—	2 doll. banco;

therefore,

As $(5.105.400.5) : 1000 :: (26.100.533.2) :$
$$\frac{1000.26.100.533.2}{5.105.400.5} = \frac{4.26.533}{21} = 2639\tfrac{13}{21}$$ dollars, the number sought.

CHAP. X.

Of Compound Relations.

488. *Compound Relations* are obtained by multiplying the terms of two or more relations, the antecedents by the antecedents, and the consequents by the consequents; we then say, that the relation between those two products is *compounded* of the relations given.

Thus the relations $a : b$, $c : d$, $e : f$, give the compound relation $ace : bdf$*.

* Each of these three *ratios* is said to be one of the *roots* of the compound ratio.

489. A relation continuing always the same, when we divide both its terms by the same number, in order to abridge it, we may greatly facilitate the above composition by comparing the antecedents and the consequents, for the purpose of making such reductions as we performed in the last chapter.

For example, we find the compound relation of the following given relations thus:

Relations given.

12 : 25, 28 : 33, and 55 : 56.

Which becomes

$$(12.28.55) : (25.33.56) = 2 : 5$$

by cancelling the common divisors.

So that 2 : 5 is the compound relation required.

490. The same operation is to be performed, when it is required to calculate generally by letters; and the most remarkable case is that in which each antecedent is equal to the consequent of the preceding relation. If the given relations are

$$a : b$$
$$b : c$$
$$c : d$$
$$d : e$$
$$e : a$$

the compound relation is 1 : 1.

491. The utility of these principles will be perceived, when it is observed, that the relation between two square fields is compounded of the relations of the lengths and the breadths.

Let the two fields, for example, be A and B; A having 500 feet in length by 60 feet in breadth; the length of B being 360 feet, and its breadth 100 feet; the relation of the lengths will be 500 : 360, and that of the breadths 60 : 100. So that we have

$$(500.60) : (360.100) = 5 : 6$$

Wherefore the field A is to the field B, as 5 to 6.

492. Again, let the field A be 720 feet long, 88 feet broad; and let the field B be 660 feet long, and 90 feet broad; the relations will be compounded in the following manner:

Relation of the lengths 720 : 660
Relation of the breadths 88 : 90

and by cancelling, the

Relation of the fields A and B is 16 : 15.

493. Farther, if it be required to compare two rooms with respect to the space, or contents, we observe, that that relation is compounded of three relations; namely, that of the lengths, breadths, and heights. Let there be, for example, a room A, whose length is 36 feet, breadth 16 feet, and height 14 feet, and a room B, whose length is 42 feet, breadth 24 feet, and height 10 feet; we shall have these three relations:

For the length	36 : 42
For the breadth	16 : 24
For the height	14 : 10

And cancelling the common measures, these become 4 : 5. So that the contents of the room A, is to the contents of the room B, as 4 to 5.

494. When the relations which we compound in this manner are equal, there result multiplicate relations. Namely, two equal relations give a *duplicate ratio*, or *ratio of the squares;* three equal relations produce the *triplicate ratio*, or *ratio of the cubes;* and so on. For example, the relations $a : b$ and $a : b$ give the compound relation $a^2 : b^2$; wherefore we say, that the squares are in the duplicate ratio of their roots. And the ratio $a : b$ multiplied twice, giving the ratio $a^3 : b^3$, we say that the cubes are in the triplicate ratio of their roots.

495. Geometry teaches, that two circular spaces are in the duplicate relation of their diameters; this means, that they are to each other as the squares of their diameters.

Let A be such a space, having its diameter 45 feet, and B another circular space, whose diameter is 30 feet; the first space will be to the second as 45×45 is to 30×30; or, compounding these two equal relations, as 9 : 4. Therefore the two areas are to each other as 9 to 4.

496. It is also demonstrated, that the solid contents of spheres are in the ratio of the cubes of their diameters: so that the diameter of a globe, A, being 1 foot, and the diameter of a globe, B, being 2 feet, the solid content of A will be to that of B, as $1^3 : 2^3$; or as 1 to 8. If, therefore, the spheres are formed of the same substance, the latter will weigh 8 times as much as the former.

497. It is evident that we may in this manner find the weight of cannon balls, their diameters, and the weight of one, being given. For example, let there be the ball A, whose diameter is 2 inches, and weight 5 pounds; and if the weight of another ball be required, whose diameter is 8 inches, we have this proportion,

$$2^3 : 8^3 :: 5 : 320 \text{ pounds},$$

which gives the weight of the ball B: and for another ball C, whose diameter is 15 inches, we should have,

$$2^3 : 15^3 :: 5 : 2109\tfrac{3}{8}\text{lb.}$$

498. When the ratio of two fractions, as $\frac{a}{b} : \frac{c}{d}$, is required, we may always express it in integer numbers; for we have only to multiply the two fractions by bd, in order to obtain the ratio $ad : bc$, which is equal to the other; and from hence results the proportion $\frac{a}{b} : \frac{c}{d} :: ad : bc$. If, therefore, ad and bc have common divisors, the ratio may be reduced to fewer terms. Thus $\frac{15}{24} : \frac{25}{36} :: (15.36) : (24.25) :: 9 : 10$.

499. If we wished to know the ratio of the fractions $\frac{1}{a}$ and $\frac{1}{b}$, it is evident that we should have $\frac{1}{a} : \frac{1}{b} :: b : a$; which is expressed by saying, that two fractions, which have unity for their numerator, are in the *reciprocal*, or *inverse* ratio of their denominators: and the same thing is said of two fractions which have any common numerator; for $\frac{c}{a} : \frac{c}{b} :: b : a$. But if two fractions have their denominators equal, as $\frac{a}{c} : \frac{b}{c}$, they are in the *direct ratio* of the numerators; namely, as $a : b$. Thus, $\frac{3}{8} : \frac{3}{16} :: \frac{6}{16} : \frac{3}{16}$, or $6 : 3 :: 2 : 1$, and $\frac{10}{7} : \frac{15}{7} :: 10 : 15$, or $2 : 3$.

500. It has been observed, in the free descent of bodies, that a body falls about 16 English feet in a second, that in two seconds of time it falls from the height of 64 feet, and in three seconds it falls 144 feet. Hence it is concluded, that the heights are to each other as the squares of the times; and, reciprocally, that the times are in the subduplicate ratio of the heights, or as the square roots of the heights *.

If, therefore, it be required to determine how long a stone will be in falling from the height of 2304 feet; we have $16 : 2304 :: 1 : 144$, the square of the time; and consequently the time required is 12 seconds.

501. If it be required to determine how far, or through

* The space, through which a heavy body descends, in the latitude of London, and in the first second of time, has been found by experiment to be $16\frac{1}{12}$ English feet; but in calculations where great accuracy is not required, the fraction may be omitted.

what height, a stone will pass by descending for the space of an hour, or 3600 seconds; we must say,

$$\text{As } 1^2 : 3600^2 :: 16 : 207360000 \text{ feet,}$$

the height required.

Which being reduced is found equal to 39272 miles; and consequently nearly five times greater than the diameter of the earth.

502. It is the same with regard to the price of precious stones, which are not sold in the proportion of their weight; every body knows that their prices follow a much greater ratio. The rule for diamonds is, that the price is in the duplicate ratio of the weight; that is to say, the ratio of the prices is equal to the square of the ratio of the weights. The weight of diamonds is expressed in carats, and a carat is equivalent to 4 grains; if, therefore, a diamond of one carat is worth 10 livres, a diamond of 100 carats will be worth as many times 10 livres as the square of 100 contains 1; so that we shall have, according to the Rule of Three,

As 1 : 10000 : : 10 : 100000 liv. *Ans.*

There is a diamond in Portugal which weighs 1680 carats; its price will be found, therefore, by making

$$1^2 : 1680^2 :: 10 : 28224000 \text{ livres.}$$

503. The posts, or mode of travelling, in France, furnish sufficient examples of compound ratios; because the price is regulated by the compound ratio of the number of horses, and the number of leagues, or posts. Thus, for example, if one horse cost 20 sous per post, it is required to find how much must be paid for 28 horses for $4\frac{1}{2}$ posts.

We write first the ratio of the horses - -	1 : 28
Under this ratio we put that of the stages -	2 : 9
And, compounding the two ratios, we have -	2 : 252

Or, by abridging the two terms, 1 : 126 : : 1 liv. to 126 fr. or 42 crowns.

Again, If I pay a ducat for eight horses for 3 miles, how much must I pay for thirty horses for four miles? The calculation is as follows:

8 : 30
3 : 4

By compounding these two ratios, and abridging,

1 : 5 : : 1 duc. : 5 ducats; the sum required.

504. The same composition occurs when workmen are to be paid, since those payments generally follow the ratio

compounded of the number of workmen and that of the days which they have been employed.

If, for example, 25 sous per day be given to one mason, and it is required what must be paid to 24 masons who have worked for 50 days, we state the calculation thus:

1 : 24
1 : 50

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1 : 1200 : : 25 : 30000 sous, or 1500 francs.

In these examples, five things being given, the rule which serves to resolve them is called, in books of arithmetic, The Rule of Five, or Double Rule of Three.

CHAP. XI.

Of Geometrical Progressions.

505. A series of numbers, which are always becoming a certain number of times greater, or less, is called a *geometrical progression*, because each term is constantly to the following one in the same geometrical ratio: and the number which expresses how many times each term is greater than the preceding, is called the *exponent*, or *ratio*. Thus, when the first term is 1 and the exponent, or ratio, is 2, the geometrical progression becomes,

Terms 1 2 3 4 5 6 7 8 9 &c.
Prog. 1, 2, 4, 8, 16, 32, 64, 128, 256, &c.

The numbers 1, 2, 3, &c. always marking the place which each term holds in the progression.

506. If we suppose, in general, the first term to be a, and the ratio b, we have the following geometrical progression:

$$1,\ 2,\ 3,\ 4,\ 5,\ 6,\ 7,\ 8\ \ldots\ n.$$
$$Prog.\ a,\ ab,\ ab^2,\ ab^3,\ ab^4,\ ab^5,\ ab^6,\ ab^7\ \ldots\ ab^{n-1}.$$

So that, when this progression consists of n terms, the last term is ab^{n-1}. We must, however, remark here, that if the ratio b be greater than unity, the terms increase continually; if $b = 1$, the terms are all equal; lastly, if b be less than 1, or a fraction, the terms continually decrease. Thus, when $a = 1$, and $b = \frac{1}{2}$, we have this geometrical progression: