Consequently, the quotient will be $4+\sqrt{2}$. The truth of this may be proved, as before, by multiplication; thus,

$$\begin{array}{l} 4+\sqrt{2} \\ 3-2\sqrt{2} \\ \hline 12+3\sqrt{2} \\ \quad -8\sqrt{2}-4 \\ \hline 12-5\sqrt{2}-4=8-5\sqrt{2}. \end{array}$$

331. In the same manner, we may transform irrational fractions into others, that have rational denominators. If we have, for example, the fraction $\frac{1}{5-2\sqrt{6}}$, and multiply its numerator and denominator by $5+2\sqrt{6}$, we transform it into this, $\frac{5+2\sqrt{6}}{1}=5+2\sqrt{6}$; in like manner, the fraction $\frac{2}{-1+\sqrt{-3}}$ assumes this form, $\frac{2+2\sqrt{-3}}{-4}=\frac{1+\sqrt{-3}}{-2}$; also $\frac{\sqrt{6}+\sqrt{5}}{\sqrt{6}-\sqrt{5}}=\frac{11+2\sqrt{30}}{1}=11+2\sqrt{30}$.

332. When the denominator contains several terms, we may, in the same manner, make the radical signs in it vanish one by one. Thus, if the fraction $\frac{1}{\sqrt{10}-\sqrt{2}-\sqrt{3}}$ be proposed, we first multiply these two terms by $\sqrt{10}+\sqrt{2}+\sqrt{3}$, and obtain the fraction $\frac{\sqrt{10}+\sqrt{2}+\sqrt{3}}{5-2\sqrt{6}}$; then multiplying its numerator and denominator by $5+2\sqrt{6}$, we have $5\sqrt{10}+11\sqrt{2}+9\sqrt{3}+2\sqrt{60}$.

CHAP. IX.

Of Cubes, *and of the* Extraction *of* Cube Roots.

333. To find the cube of $a+b$, we have only to multiply its square, $a^2+2ab+b^2$, again by $a+b$, thus;

$$\begin{array}{l} a^2+2ab+b^2 \\ a+b \\ \hline a^3+2a^2b+ab^2 \\ \quad\quad a^2b+2ab^2+b^3 \\ \hline \end{array}$$

and the cube will be $a^3+3a^2b+3ab^2+b^3$.

We see therefore that it contains the cubes of the two parts of the root, and, beside that, $3a^2b + 3ab^2$; which quantity is equal to $(3ab) \times (a + b)$; that is, the triple product of the two parts, a and b, multiplied by their sum.

334. So that whenever a root is composed of two terms, it is easy to find its cube by this rule: for example, the number $5=3+2$; its cube is therefore $27+8+(18\times 5) = 125$.

And if $7 + 3 = 10$ be the root; then the cube will be $343 + 27 + (63 \times 10) = 1000$.

To find the cube of 36, let us suppose the root $36 = 30 + 6$, and we have for the cube required, $27000 + 216 + (540 \times 36) = 46656$.

335. But if, on the other hand, the cube be given, namely, $a^3 + 3a^2b + 3ab^2 + b^3$, and it be required to find its root, we must premise the following remarks:

First, when the cube is arranged according to the powers of one letter, we easily know by the leading term a^3, the first term a of the root, since the cube of it is a^3; if, therefore, we subtract that cube from the cube proposed, we obtain the remainder, $3a^2b + 3ab^2 + b^3$, which must furnish the second term of the root.

336. But as we already know, from Art. 333, that the second term is $+b$, we have principally to discover how it may be derived from the above remainder. Now, that remainder may be expressed by two factors, thus, $(3a^2 + 3ab + b^2) \times (b)$; if, therefore, we divide by $3a^2 + 3ab + b^2$, we obtain the second part of the root $+b$, which is required.

337. But as this second term is supposed to be unknown, the divisor also is unknown; nevertheless we have the first term of that divisor, which is sufficient: for it is $3a^2$, that is, thrice the square of the first term already found; and by means of this, it is not difficult to find also the other part, b, and then to complete the divisor before we perform the division; for this purpose, it will be necessary to join to $3a^2$ thrice the product of the two terms, or $3ab$, and b^2, or the square of the second term of the root.

338. Let us apply what we have said to two examples of other given cubes.

$$
\begin{array}{rl}
 & a^3+12a^2+48a+64 \;\; (a+4 \\
 & a^3 \\
\hline
3a^2+12a+16) & 12a^2+48a+64 \\
 & 12a^2+48a+64 \\
\hline
 & 0.
\end{array}
$$

$$\begin{array}{rl}
 & a^6 - 6a^5 + 15a^4 - 20a^3 + 15a^2 - 6a + 1\,(a^2 - 2a + 1 \\
 & a^6 \\ \hline
3a^4 - 6a^3 + 4a^2) & -6a^5 + 15a^4 - 20a^3 \\
 & -6a^5 + 12a^4 - 8a^3 \\ \hline
3a^4 - 12a^3 + 12a^2 + 3a^2 - 6a + 1\,) & 3a^4 - 12a^3 + 15a^2 - 6a + 1 \\
 & 3a^4 - 12a^3 + 15a^2 - 6a + 1 \\ \hline
 & 0.
\end{array}$$

339. The analysis which we have given is the foundation of the common rule for the extraction of the cube root in numbers. See the following example of the operation in the number 2197:

$$\begin{array}{r|l}
 & \dot{2}19\dot{7}(10 + 3 = 13 \\
 & 1000 \\ \hline
300 & 1197 \\
90 & \\
9 & \\ \hline
399 & 1197 \\ \hline
 & 0.
\end{array}$$

Let us also extract the cube root of 34965783:

$$\begin{array}{r|l}
 & \dot{3}4965\dot{7}8\dot{3}(300 + 20 + 7, \text{ or } 327 \\
 & 27000000 \\ \hline
270000 & 7965783 \\
18000 & \\
400 & \\ \hline
288400 & 5768000 \\ \hline
307200 & 2197783 \\
6720 & \\
49 & \\ \hline
313969 & 2197783 \\ \hline
 & 0.
\end{array}$$