4. Reduce $\frac{x^2-y^2}{x^4-y^4}$ to its lowest terms. *Ans.* $\frac{1}{x^2+y^2}$.

5. Reduce $\frac{a^4-x^4}{a^3-a^2x-ax^2+x^3}$ to its lowest terms.

Ans. $\frac{a^2+x^2}{a-x}$.

6. Reduce $\frac{5a^5+10a^4x+5a^3x^2}{a^3x+2a^2x^2+2ax^3+x^4}$ to its lowest terms.

Ans. $\frac{5a^4+5a^3x}{a^2x+ax^2+x^3}$

CHAP. IX.

Of the Addition *and* Subtraction *of* Fractions.

94. When fractions have equal denominators, there is no difficulty in adding and subtracting them; for $\frac{2}{7}+\frac{3}{7}$ is equal to $\frac{5}{7}$, and $\frac{4}{7}-\frac{2}{7}$ is equal to $\frac{2}{7}$. In this case, therefore, either for addition or subtraction, we alter only the numerators, and place the common denominator under the line, thus;

$\frac{7}{100}+\frac{9}{100}-\frac{12}{100}-\frac{15}{100}+\frac{20}{100}$ is equal to $\frac{9}{100}$;
$\frac{24}{50}-\frac{7}{50}-\frac{12}{50}+\frac{31}{50}$ is equal to $\frac{36}{50}$, or $\frac{18}{25}$;
$\frac{16}{20}-\frac{3}{20}-\frac{11}{20}+\frac{14}{20}$ is equal to $\frac{16}{20}$, or $\frac{4}{5}$;

also $\frac{1}{3}+\frac{2}{3}$ is equal to $\frac{3}{3}$, or 1, that is to say, an integer; and $\frac{2}{4}-\frac{3}{4}+\frac{1}{4}$ is equal to $\frac{0}{4}$, that is to say, nothing, or 0.

95. But when fractions have not equal denominators, we can always change them into other fractions that have the same denominator. For example, when it is proposed to add together the fractions $\frac{1}{2}$ and $\frac{1}{3}$, we must consider that $\frac{1}{2}$ is the same as $\frac{3}{6}$, and that $\frac{1}{3}$ is equivalent to $\frac{2}{6}$; we have therefore, instead of the two fractions proposed, $\frac{3}{6}+\frac{2}{6}$, the sum of which is $\frac{5}{6}$. And if the two fractions were united by the sign *minus* as $\frac{1}{2}-\frac{1}{3}$, we should have $\frac{3}{6}-\frac{2}{6}$, or $\frac{1}{6}$.

As another example, let the fractions proposed be $\frac{3}{4}+\frac{5}{8}$. Here, since $\frac{3}{4}$ is the same as $\frac{6}{8}$, this value may be substituted for $\frac{3}{4}$, and we may then say $\frac{6}{8}+\frac{5}{8}$ makes $\frac{11}{8}$, or $1\frac{3}{8}$.

Suppose farther, that the sum of $\frac{1}{3}$ and $\frac{1}{4}$ were required, I say that it is $\frac{7}{12}$; for $\frac{1}{3}=\frac{4}{12}$, and $\frac{1}{4}=\frac{3}{12}$: therefore $\frac{4}{12}+\frac{3}{12}=\frac{7}{12}$.

96. We may have a greater number of fractions to reduce

to a common denominator; for example, $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$, $\frac{5}{6}$. In this case, the whole depends on finding a number that shall be divisible by all the denominators of those fractions. In this instance, 60 is the number which has that property, and which consequently becomes the common denominator. We shall therefore have $\frac{30}{60}$, instead of $\frac{1}{2}$; $\frac{40}{60}$, instead of $\frac{2}{3}$; $\frac{45}{60}$, instead of $\frac{3}{4}$; $\frac{48}{60}$, instead of $\frac{4}{5}$; and $\frac{50}{60}$, instead of $\frac{5}{6}$. If now it be required to add together all these fractions $\frac{30}{60}$, $\frac{40}{60}$, $\frac{45}{60}$, $\frac{48}{60}$, and $\frac{50}{60}$; we have only to add all the numerators, and under the sum place the common denominator 60; that is to say, we shall have $\frac{213}{60}$, or 3 integers, and the fractional remainder $\frac{11}{20}$.

97. The whole of this operation consists, as we before stated, in changing fractions, whose denominators are unequal, into others whose denominators are equal. In order, therefore, to perform it generally, let $\frac{a}{b}$ and $\frac{c}{d}$ be the fractions proposed. First, multiply the two terms of the first fraction by d, and we shall have the fraction $\frac{ad}{bd}$ equal to $\frac{a}{b}$; next multiply the two terms of the second fraction by b, and we shall have an equivalent value of it expressed by $\frac{bc}{bd}$; thus the two denominators are become equal. Now, if the sum of the two proposed fractions be required, we may immediately answer that it is $\frac{ad+bc}{bd}$; and if their difference be asked, we say that it is $\frac{ad-bc}{bd}$. If the fractions $\frac{5}{8}$ and $\frac{7}{9}$, for example, were proposed, we should obtain, in their stead, $\frac{45}{72}$ and $\frac{56}{72}$; of which the sum is $\frac{101}{72}$ and the difference $\frac{11}{72}$*.

98. To this part of the subject belongs also the question, Of two proposed fractions which is the greater or the less?

* The rule for reducing fractions to a common denominator may be concisely expressed thus. Multiply each numerator into every denominator except its own, for a new numerator, and all the denominators together for the common denominator. When this operation has been performed, it will appear that the numerator and denominator of each fraction have been multiplied by the same quantity, and consequently retain the same value.

for, to resolve this, we have only to reduce the two fractions to the same denominator. Let us take, for example, the two fractions $\frac{2}{3}$ and $\frac{5}{7}$; when reduced to the same denominator, the first becomes $\frac{14}{21}$, and the second $\frac{15}{21}$, where it is evident that the second, or $\frac{5}{7}$, is the greater, and exceeds the former by $\frac{1}{21}$.

Again, if the fractions $\frac{3}{5}$ and $\frac{5}{8}$ be proposed, we shall have to substitute for them $\frac{24}{40}$ and $\frac{25}{40}$; whence we may conclude that $\frac{5}{8}$ exceeds $\frac{3}{5}$, but only by $\frac{1}{40}$.

99. When it is required to subtract a fraction from an integer, it is sufficient to change one of the units of that integer into a fraction, which has the same denominator as that which is to be subtracted; then in the rest of the operation there is no difficulty. If it be required, for example, to subtract $\frac{2}{3}$ from 1, we write $\frac{3}{3}$ instead of 1, and say that $\frac{2}{3}$ taken from $\frac{3}{3}$ leaves the remainder $\frac{1}{3}$. So $\frac{5}{12}$, subtracted from 1, leaves $\frac{7}{12}$.

If it were required to subtract $\frac{3}{4}$ from 2, we should write 1 and $\frac{4}{4}$ instead of 2, and should then immediately see that after the subtraction there must remain $1\frac{1}{4}$.

100. It happens also sometimes, that having added two or more fractions together, we obtain more than an integer; that is to say, a numerator greater than the denominator: this is a case which has already occurred, and deserves attention.

We found, for example [Article 96], that the sum of the five fractions $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$, and $\frac{5}{6}$ was $\frac{213}{60}$, and remarked that the value of this sum was $3\frac{33}{60}$ or $3\frac{11}{20}$. Likewise, $\frac{2}{3}+\frac{3}{4}$, or $\frac{8}{12}+\frac{9}{12}$, makes $\frac{17}{12}$, or $1\frac{5}{12}$. We have therefore only to perform the actual division of the numerator by the denominator, to see how many integers there are for the quotient, and to set down the remainder.

Nearly the same must be done to add together numbers compounded of integers and fractions; we first add the fractions, and if the sum produces one or more integers, these are added to the other integers. If it be proposed, for example, to add $3\frac{1}{2}$ and $2\frac{2}{3}$; we first take the sum of $\frac{1}{2}$ and $\frac{2}{3}$, or of $\frac{3}{6}$ and $\frac{4}{6}$, which is $\frac{7}{6}$, or $1\frac{1}{6}$; and thus we find the total sum to be $6\frac{1}{6}$.

QUESTIONS FOR PRACTICE.

1. Reduce $\frac{2x}{a}$ and $\frac{b}{c}$ to a common denominator.

Ans. $\frac{2cx}{ac}$ and $\frac{ab}{ac}$.

2. Reduce $\frac{a}{b}$ and $\frac{a+b}{c}$ to a common denominator.

Ans. $\frac{ac}{bc}$ and $\frac{ab+b^2}{bc}$.

3. Reduce $\frac{3x}{2a}$, $\frac{2b}{3c}$, and d to fractions having a common denominator.

Ans. $\frac{9cx}{6ac}$, $\frac{4ab}{6ac}$ and $\frac{6acd}{6ac}$.

4. Reduce $\frac{3}{4}$, $\frac{2x}{3}$ and $a+\frac{2x}{a}$ to a common denominator.

Ans. $\frac{9a}{12a}$, $\frac{8ax}{12a}$, and $\frac{12a^2+24x}{12a}$.

5. Reduce $\frac{1}{2}$, $\frac{a^2}{3}$, and $\frac{x^2+a^2}{x+a}$, to a common denominator.

Ans. $\frac{3x+3a}{6x+6a}$, $\frac{2a^2x+2a^3}{6x+6a}$, $\frac{6x^2+6a^2}{6x+6a}$.

6. Reduce $\frac{b}{2a^2}$, $\frac{c}{2a}$, and $\frac{d}{a}$, to a common denominator.

Ans. $\frac{2a^2b}{4a^4}$, $\frac{2a^3c}{4a^4}$, and $\frac{4a^3d}{4a^4}$; or $\frac{b}{2a^2}$, $\frac{ac}{2a^2}$, and $\frac{2ad}{2a^2}$.

CHAP. X.

Of the Multiplication *and* Division *of* Fractions.

101. The rule for the multiplication of a fraction by an integer, or whole number, is to multiply the numerator only by the given number, and not to change the denominator: thus,

2 times, or twice $\frac{1}{2}$ makes $\frac{2}{2}$, or 1 integer;
2 times, or twice $\frac{1}{3}$ makes $\frac{2}{3}$; and
3 times, or thrice $\frac{1}{6}$ makes $\frac{3}{6}$, or $\frac{1}{2}$;
4 times $\frac{5}{12}$ makes $\frac{20}{12}$, or $1\frac{8}{12}$, or $1\frac{2}{3}$.

But, instead of this rule, we may use that of dividing the denominator by the given integer, which is preferable, when it can be done, because it shortens the operation. Let it be required, for example, to multiply $\frac{8}{9}$ by 3; if we multiply the numerator by the given integer we obtain $\frac{24}{9}$, which