thus 0, or nothing, will always be the value of $+ a - a$; but if we wish to know the value of $+ a - b$, two cases are to be considered.

The first is when a is greater than b; b must then be subtracted from a, and the remainder (before which is placed, or understood to be placed, the sign $+$) shews the value sought.

The second case is that in which a is less than b: here a is to be subtracted from b, and the remainder being made negative, by placing before it the sign $-$, will be the value sought.

CHAP. III.

Of the Multiplication *of* Simple Quantities.

23. When there are two or more equal numbers to be added together, the expression of their sum may be abridged: for example,

$a + a$ is the same with $2 \times a$,
$a + a + a$ - - - - - $3 \times a$,
$a + a + a + a$ - - - $4 \times a$, and so on; where $\times$ is the sign of multiplication. In this manner we may form an idea of multiplication; and it is to be observed that,

$2 \times a$ signifies 2 times, or twice a
$3 \times a$ - - - - 3 times, or thrice a
$4 \times a$ - - - - 4 times a, &c.

24. If therefore a number expressed by a letter is to be multiplied by any other number, we simply put that number before the letter, thus;

a multiplied by 20 is expressed by $20a$, and
b multiplied by 30 is expressed by $30b$, &c.

It is evident, also, that c taken once, or $1c$, is the same as c.

25. Farther, it is extremely easy to multiply such products again by other numbers; for example:

2 times, or twice $3a$, makes $6a$
3 times, or thrice $4b$, makes $12b$
5 times $7x$ makes $35x$.

and these products may be still multiplied by other numbers at pleasure.

26. When the number by which we are to multiply is also represented by a letter, we place it immediately before the other letter; thus, in multiplying b by a, the product is

written ab; and pq will be the product of the multiplication of the number q by p. Also, if we multiply this pq again by a, we shall obtain apq.

27. It may be farther remarked here, that the order in which the letters are joined together is indifferent; thus ab is the same thing as ba; for b multiplied by a is the same as a multiplied by b. To understand this, we have only to substitute, for a and b, known numbers, as 3 and 4; and the truth will be self-evident; for 3 times 4 is the same as 4 times 3.

28. It will not be difficult to perceive, that when we substitute numbers for letters joined together, in the manner we have described, they cannot be written in the same way by putting them one after the other. For if we were to write 34 for 3 times 4, we should have 34 and not 12. When therefore it is required to multiply common numbers, we must separate them by the sign $\times$, or by a point: thus, 3×4, or 3.4, signifies 3 times 4; that is, 12. So, 1×2 is equal to 2; and $1 \times 2 \times 3$ makes 6. In like manner, $1 \times 2 \times 3 \times 4 \times 56$ makes 1344; and $1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 \times 9 \times 10$ is equal to 3628800, &c.

29. In the same manner we may discover the value of an expression of this form, $5.7.8.abcd$. It shews that 5 must be multiplied by 7, and that this product is to be again multiplied by 8; that we are then to multiply this product of the three numbers by a, next by b, then by c, and lastly by d. It may be observed, also, that instead of 5.7.8, we may write its value, 280; for we obtain this number when we multiply the product of 5 by 7, or 35, by 8.

30. The results which arise from the multiplication of two or more numbers are called *products;* and the numbers, or individual letters, are called *factors.*

31. Hitherto we have considered only positive numbers, and there can be no doubt, but that the products which we have seen arise are positive also: viz. $+a$ by $+b$ must necessarily give $+ab$. But we must separately examine what the multiplication of $+a$ by $-b$, and of $-a$ by $-b$, will produce.

32. Let us begin by multiplying $-a$ by 3 or $+3$. Now, since $-a$ may be considered as a debt, it is evident that if we take that debt three times, it must thus become three times greater, and consequently the required product is $-3a$. So if we multiply $-a$ by $+b$, we shall obtain $-ba$, or, which is the same thing, $-ab$. Hence we conclude, that if a positive quantity be multiplied by a negative quantity, the product will be negative; and it may be laid down

as a rule, that + by + makes + or *plus;* and that, on the contrary, + by −, or − by +, gives −, or *minus.*

33. It remains to resolve the case in which − is multiplied by − ; or, for example, $-a$ by $-b$. It is evident, at first sight, with regard to the letters, that the product will be ab; but it is doubtful whether the sign +, or the sign −, is to be placed before it; all we know is, that it must be one or the other of these signs. Now, I say that it cannot be the sign − : for $-a$ by $+b$ gives $-ab$, and $-a$ by $-b$ cannot produce the same result as $-a$ by $+b$; but must produce a contrary result, that is to say, $+ab$; consequently, we have the following rule: − multiplied by − produces +, that is, the same as + multiplied by + *.

* A further illustration of this rule is generally given by algebraists as follows:

First, we know that $+a$ multiplied by $+b$ gives the product $+ab$; and if $+a$ be multiplied by a quantity less than b, as $b-c$, the product must necessarily be less than ab; in short, from ab we must subtract the product of a, multiplied by c; hence $a \times (b-c)$ must be expressed by $ab-ac$; therefore it follows that $a \times -c$ gives the product $-ac$.

If now we consider the product arising from the multiplication of the two quantities $(a-b)$, and $(c-d)$, we know that it is less than that of $(a-b) \times c$, or of $ac-bc$; in short, from this product we must subtract that of $(a-b) \times d$; but the product $(a-b) \times (c-d)$ becomes $ac-bc-ad$, together with the product of $-b \times -d$ annexed; and the question is only what sign we must employ for this purpose, whether + or −. Now we have seen that from the product $ac-bc$ we must subtract the product of $(a-b) \times d$, that is, we must subtract a quantity less than ad; we have therefore subtracted already too much by the quantity bd; this product must therefore be added; that is, it must have the sign + prefixed; hence we see that $-b \times -d$ gives $+bd$ for a product; or − *minus* multiplied by − *minus* gives + *plus*. See Art. 273, 274.

Multiplication has been erroneously called a compendious method of performing addition: whereas it is the taking, or repeating of one given number as many times, as the number by which it is to be multiplied, contains units. Thus, 9×3 means that 9 is to be taken 3 times, or that the measure of multiplication is 3; again, $9 \times \frac{1}{2}$ means that 9 is to be taken half a time, or that the measure of multiplication is $\frac{1}{2}$. In multiplication there are two factors, which are sometimes called the multiplicand and the multiplier. These, it is evident, may reciprocally change places, and the product will be still the same: for $9 \times 3 = 3 \times 9$, and $9 \times \frac{1}{2} = \frac{1}{2} \times 9$. Hence it appears, that numbers may be diminished by multiplication, as well as increased, in any given ratio, which is wholly inconsistent with

34. The rules which we have explained are expressed more briefly as follows:

Like signs, multiplied together, give $+$; unlike or contrary signs give $-$. Thus, when it is required to multiply the following numbers; $+a$, $-b$, $-c$, $+d$; we have first $+a$ multiplied by $-b$, which makes $-ab$; this by $-c$, gives $+abc$; and this by $+d$, gives $+abcd$.

35. The difficulties with respect to the signs being removed, we have only to shew how to multiply numbers that are themselves products. If we were, for instance, to multiply the number ab by the number cd, the product would be $abcd$, and it is obtained by multiplying first ab by c, and then the result of that multiplication by d. Or, if we had to multiply 36 by 12; since 12 is equal to 3 times 4, we

the nature of Addition; for $9 \times \frac{1}{2} = 4\frac{1}{2}$, $9 \times \frac{1}{9} = 1$, $9 \times \frac{1}{900} = \frac{1}{100}$, &c. The same will be found true with respect to algebraic quantities; $a \times b = ab$, $-9 \times 3 = -27$, that is, 9 negative integers multiplied by 3, or taken 3 times, are equal to -27, because the measure of multiplication is 3. In the same manner, by inverting the factors, 3 positive integers multiplied by -9, or taken 9 times negatively, must give the same result. Therefore a positive quantity taken negatively, or a negative quantity taken positively, gives a negative product.

From these considerations, we may illustrate the present subject in a different way, and shew, that the product of two negative quantities must be positive. First, algebraic quantities may be considered as a series of numbers increasing in any ratio, on each side of nothing, to infinity. Let us assume a small part only of such a series for the present purpose, in which the ratio is unity, and let us multiply every term of it by -2.

$$\begin{array}{rrrrrrrrrrr} 5, & 4, & 3, & 2, & 1, & 0, & -1, & -2, & -3, & -4, & -5, \\ -2, & -2, & -2, & -2, & -2, & -2, & -2, & -2, & -2, & -2, & -2, \\ \hline -10, & -8, & -6, & -4, & -2, & 0, & +2, & +4, & +6, & +8, & +10. \end{array}$$

Here, of course, we find the series inverted, and the ratio doubled. Farther, in order to illustrate the subject, we may consider the ratio of a series of fractions between 1 and 0, as indefinitely small, till the last term being multiplied by -2, the product would be equal to 0. If, after this, the multiplier having passed the middle term, 0, be multiplied into any negative term, however small, between 0 and -1, on the other side of the series, the product, it is evident, must be positive, otherwise the series could not go on. Hence it appears, that the taking of a negative quantity negatively destroys the very property of negation, and is the conversion of negative into positive numbers. So that if $+ \times - = -$, it necessarily follows that $- \times -$ must give a contrary product, that is, $+$. See Art. 176, 177.

should only multiply 36 first by 3, and then the product 108 by 4, in order to have the whole product of the multiplication of 12 by 36, which is consequently 432.

36. But if we wished to multiply $5ab$ by $3cd$, we might write $3cd \times 5ab$. However, as in the present instance the order of the numbers to be multiplied is indifferent, it will be better, as is also the custom, to place the common numbers before the letters, and to express the product thus: $5 \times 3abcd$, or $15abcd$; since 5 times 3 is 15.

So if we had to multiply $12pqr$ by $7xy$, we should obtain $12 \times 7pqrxy$, or $84pqrxy$.

CHAP. IV.

Of the Nature of whole Numbers, *or* Integers, *with respect to their* Factors.

37. We have observed that a product is generated by the multiplication of two or more numbers together, and that these numbers are called *factors*. Thus, the numbers a, b, c, d, are the factors of the product $abcd$.

38. If, therefore, we consider all whole numbers as products of two or more numbers multiplied together, we shall soon find that some of them cannot result from such a multiplication, and consequently have not any factors; while others may be the products of two or more numbers multiplied together, and may consequently have two or more factors. Thus 4 is produced by 2×2; 6 by 2×3; 8 by $2 \times 2 \times 2$; 27 by $3 \times 3 \times 3$; and 10 by 2×5, &c.

39. But on the other hand, the numbers 2, 3, 5, 7, 11, 13, 17, &c. cannot be represented in the same manner by factors, unless for that purpose we make use of unity, and represent 2, for instance, by 1×2. But the numbers which are multiplied by 1 remaining the same, it is not proper to reckon unity as a factor.

All numbers, therefore, such as 2, 3, 5, 7, 11, 13, 17, &c. which cannot be represented by factors, are called *simple*, or *prime numbers*; whereas others, as 4, 6, 8, 9, 10, 12, 14, 15, 16, 18, &c. which may be represented by factors, are called *composite numbers*.

40. *Simple* or *prime numbers* deserve therefore particular attention, since they do not result from the mul-