CHAP. II.

Explanation of the Signs + Plus *and* − Minus.

8. When we have to add one given number to another, this is indicated by the sign +, which is placed before the second number, and is read *plus*. Thus 5 + 3 signifies that we must add 3 to the number 5, in which case, every one knows that the result is 8; in the same manner 12 + 7 make 19; 25 + 16 make 41; the sum of 25 + 41 is 66, &c.

9. We also make use of the same sign + *plus*, to connect several numbers together; for example, 7 + 5 + 9 signifies that to the number 7 we must add 5, and also 9, which make 21. The reader will therefore understand what is meant by

$$8 + 5 + 13 + 11 + 1 + 3 + 10,$$

viz. the sum of all these numbers, which is 51.

10. All this is evident; and we have only to mention, that in Algebra, in order to generalise numbers, we represent them by letters, as a, b, c, d, &c. Thus, the expression $a + b$, signifies the sum of two numbers, which we express by a and b, and these numbers may be either very great, or very small. In the same manner, $f + m + b + x$, signifies the sum of the numbers represented by these four letters.

If we know therefore the numbers that are represented by letters, we shall at all times be able to find, by arithmetic, the sum or value of such expressions.

11. When it is required, on the contrary, to subtract one given number from another, this operation is denoted by the sign −, which signifies *minus*, and is placed before the number to be subtracted: thus, 8 − 5 signifies that the number 5 is to be taken from the number 8; which being done, there remain 3. In like manner 12 − 7 is the same as 5; and 20 − 14 is the same as 6, &c.

12. Sometimes also we may have several numbers to subtract from a single one; as, for instance, 50 − 1 − 3 − 5 − 7 − 9. This signifies, first, take 1 from 50, and there remain 49; take 3 from that remainder, and there will remain 46; take away 5, and 41 remain; take away 7, and 34 remain; lastly, from that take 9, and there remain 25: this last remainder is the value of the expression. But as the numbers 1, 3, 5, 7, 9, are all to be subtracted, it is the

same thing if we subtract their sum, which is 25, at once from 50, and the remainder will be 25 as before.

13. It is also easy to determine the value of similar expressions, in which both the signs + *plus* and — *minus* are found. For example;

$$12 - 3 - 5 + 2 - 1 \text{ is the same as } 5.$$

We have only to collect separately the sum of the numbers that have the sign + before them, and subtract from it the sum of those that have the sign —. Thus, the sum of 12 and 2 is 14; and that of 3, 5, and 1, is 9; hence 9 being taken from 14, there remain 5.

14. It will be perceived, from these examples, that the order in which we write the numbers is perfectly indifferent and arbitrary, provided the proper sign of each be preserved. We might with equal propriety have arranged the expression in the preceding article thus; $12 + 2 - 5 - 3 - 1$, or $2 - 1 - 3 - 5 + 12$, or $2 + 12 - 3 - 1 - 5$, or in still different orders; where it must be observed, that in the arrangement first proposed, the sign + is supposed to be placed before the number 12.

15. It will not be attended with any more difficulty if, in order to generalise these operations, we make use of letters instead of real numbers. It is evident, for example, that

$$a - b - c + d - e,$$

signifies that we have numbers expressed by a and d, and that from these numbers, or from their sum, we must subtract the numbers expressed by the letters b, c, e, which have before them the sign —.

16. Hence it is absolutely necessary to consider what sign is prefixed to each number: for in Algebra, simple quantities are numbers considered with regard to the signs which precede, or affect them. Farther, we call those *positive quantities*, before which the sign + is found; and those are called *negative quantities*, which are affected by the sign —.

17. The manner in which we generally calculate a person's property, is an apt illustration of what has just been said. For we denote what a man really possesses by positive numbers, using, or understanding the sign +; whereas his debts are represented by negative numbers, or by using the sign —. Thus, when it is said of any one that he has 100 crowns, but owes 50, this means that his real possession amounts to $100 - 50$; or, which is the same thing, $+ 100 - 50$, that is to say, 50.

18. Since negative numbers may be considered as debts, because positive numbers represent real possessions, we

may say that negative numbers are less than nothing. Thus, when a man has nothing of his own, and owes 50 crowns, it is certain that he has 50 crowns less than nothing; for if any one were to make him a present of 50 crowns to pay his debts, he would still be only at the point nothing, though really richer than before.

19. In the same manner, therefore, as positive numbers are incontestably greater than nothing, negative numbers are less than nothing. Now, we obtain positive numbers by adding 1 to 0, that is to say; 1 to nothing; and by continuing always to increase thus from unity. This is the origin of the series of numbers called *natural numbers;* the following being the leading terms of this series:

$0, +1, +2, +3, +4, +5, +6, +7, +8, +9, +10,$
and so on to infinity.

But if, instead of continuing this series by successive additions, we continued it in the opposite direction, by perpetually subtracting unity, we should have the following series of negative numbers:

$0, -1, -2, -3, -4, -5, -6, -7, -8, -9, -10,$
and so on to infinity.

20. All these numbers, whether positive or negative, have the known appellation of whole numbers, or *integers,* which consequently are either greater or less than nothing. We call them *integers,* to distinguish them from fractions, and from several other kinds of numbers, of which we shall hereafter speak. For instance, 50 being greater by an entire unit than 49, it is easy to comprehend that there may be, between 49 and 50, an infinity of intermediate numbers, all greater than 49, and yet all less than 50. We need only imagine two lines, one 50 feet, the other 49 feet long, and it is evident that an infinite number of lines may be drawn, all longer than 49 feet, and yet shorter than 50.

21. It is of the utmost importance through the whole of Algebra, that a precise idea should be formed of those negative quantities, about which we have been speaking. I shall, however, content myself with remarking here, that all such expressions as

$$+1-1,\ +2-2,\ +3-3,\ +4-4, \text{\&c.}$$

are equal to 0, or nothing. And that

$$+2-5 \text{ is equal to } -3:$$

for if a person has 2 crowns, and owes 5, he has not only nothing, but still owes 3 crowns. In the same manner,

$7-12$ is equal to -5, and $25-40$ is equal to -15.

22. The same observations hold true, when, to make the expression more general, letters are used instead of numbers;

thus 0, or nothing, will always be the value of $+ a - a$; but if we wish to know the value of $+ a - b$, two cases are to be considered.

The first is when a is greater than b; b must then be subtracted from a, and the remainder (before which is placed, or understood to be placed, the sign $+$) shews the value sought.

The second case is that in which a is less than b: here a is to be subtracted from b, and the remainder being made negative, by placing before it the sign $-$, will be the value sought.

CHAP. III.

Of the Multiplication *of* Simple Quantities.

23. When there are two or more equal numbers to be added together, the expression of their sum may be abridged: for example,

$a + a$ is the same with $2 \times a$,
$a + a + a$ - - - - - $3 \times a$,
$a + a + a + a$ - - - $4 \times a$, and so on; where $\times$ is the sign of multiplication. In this manner we may form an idea of multiplication; and it is to be observed that,

$2 \times a$ signifies 2 times, or twice a
$3 \times a$ - - - - 3 times, or thrice a
$4 \times a$ - - - - 4 times a, &c.

24. If therefore a number expressed by a letter is to be multiplied by any other number, we simply put that number before the letter, thus;

a multiplied by 20 is expressed by $20a$, and
b multiplied by 30 is expressed by $30b$, &c.

It is evident, also, that c taken once, or $1c$, is the same as c.

25. Farther, it is extremely easy to multiply such products again by other numbers; for example:

2 times, or twice $3a$, makes $6a$
3 times, or thrice $4b$, makes $12b$
5 times $7x$ makes $35x$.

and these products may be still multiplied by other numbers at pleasure.

26. When the number by which we are to multiply is also represented by a letter, we place it immediately before the other letter; thus, in multiplying b by a, the product is