the method of finding the powers of powers, this being done by multiplication. When we seek, for example, the square, or the second power of a^3, we find a^6; and in the same manner we find a^{12} for the third power, or the cube, of a^4. To obtain the square of a power, we have only to double its exponent; for its cube, we must triple the exponent; and so on. Thus, the square of a^n is a^{2n}; the cube of a^n is a^{3n}; the seventh power of a^n is a^{7n}, &c.

188. The square of a^2, or the square of the square of a, being a^4, we see why the fourth power is called the *biquadrate:* also, the square of a^3 being a^6, the sixth power has received the name of *the square-cubed.*

Lastly, the cube of a^3 being a^9, we call the ninth power the *cubo-cube:* after this, no other denominations of this kind have been introduced for powers; and, indeed, the two last are very little used.

CHAP. XVIII.

Of Roots, *with relation to* Powers *in general.*

189. Since the square root of a given number is a number, whose square is equal to that given number; and since the cube root of a given number is a number, whose cube is equal to that given number; it follows that any number whatever being given, we may always suppose such roots of it, that the fourth, or the fifth, or any other power of them, respectively, may be equal to the given number. To distinguish these different kinds of roots better, we shall call the square root, *the second root;* and the cube root, *the third root;* because according to this denomination we may call *the fourth root,* that whose biquadrate is equal to a given number; and *the fifth root,* that whose fifth power is equal to a given number, &c.

190. As the square, or second root, is marked by the sign $\sqrt{}$, and the cubic, or third root, by the sign $\sqrt[3]{}$, so the fourth root is represented by the sign $\sqrt[4]{}$; the fifth root by the sign $\sqrt[5]{}$; and so on. It is evident that, according to this method of expression, the sign of the square root ought to be $\sqrt[2]{}$: but as of all roots this occurs most frequently, it has been agreed, for the sake of brevity, to omit the number 2 as the sign of this root. So that when the radical sign has no num

ber prefixed to it, this always shews that the square root is meant.

191. To explain this matter still better, we shall here exhibit the different roots of the number a, with their respective values:

$$\left.\begin{array}{l}\sqrt{a}\\ \sqrt[3]{a}\\ \sqrt[4]{a}\\ \sqrt[5]{a}\\ \sqrt[6]{a}\end{array}\right\}\text{ is the }\left\{\begin{array}{l}\text{2d}\\ \text{3d}\\ \text{4th}\\ \text{5th}\\ \text{6th}\end{array}\right\}\text{ root of }\left\{\begin{array}{l}a,\\ a,\\ a,\\ a,\\ a,\text{ and so on.}\end{array}\right.$$

So that, conversely,

$$\left.\begin{array}{l}\text{The 2d}\\ \text{The 3d}\\ \text{The 4th}\\ \text{The 5th}\\ \text{The 6th}\end{array}\right\}\text{ power of }\left\{\begin{array}{l}\sqrt{a}\\ \sqrt[3]{a}\\ \sqrt[4]{a}\\ \sqrt[5]{a}\\ \sqrt[6]{a}\end{array}\right\}\text{ is equal to }\left\{\begin{array}{l}a,\\ a,\\ a,\\ a,\\ a,\text{ and so on.}\end{array}\right.$$

192. Whether the number a therefore be great or small, we know what value to affix to all these roots of different degrees.

It must be remarked also, that if we substitute unity for a, all those roots remain constantly 1; because all the powers of 1 have unity for their value. If the number a be greater than 1, all its roots will also exceed unity. Lastly, if that number be less than 1, all its roots will also be less than unity.

193. When the number a is positive, we know from what was before said of the square and cube roots, that all the other roots may also be determined, and will be real and possible numbers.

But if the number a be negative, its second, fourth, sixth, and all its even roots, become impossible, or imaginary numbers; because all the powers of an even order, whether of positive or of negative numbers, are affected by the sign $+$: whereas the third, fifth, seventh, and all its odd roots, become negative, but rational; because the odd powers of negative numbers are also negative.

194. We have here also an inexhaustible source of new kinds of surds, or irrational quantities; for whenever the number a is not really such a power, as some one of the foregoing indices represents, or seems to require, it is impossible to express that root either in whole numbers or in fractions; and, consequently, it must be classed among the numbers which are called irrational.