179. Lastly, we have to consider the powers of negative numbers. Suppose the given number to be $-a$; then its powers will form the following series:

$$-a, \ +a^2, \ -a^3, \ +a^4, \ -a^5, \ +a^6, \ \&c,$$

Where we may observe, that those powers only become negative, whose exponents are odd numbers, and that, on the contrary, all the powers, which have an even number for the exponent, are positive. So that the third, fifth, seventh, ninth, &c. powers have all the sign $-$; and the second, fourth, sixth, eighth, &c. powers are affected by the sign $+$.

CHAP. XVII.

Of the Calculation *of* Powers.

180. We have nothing particular to observe with regard to the *Addition* and *Subtraction* of powers; for we only represent those operations by means of the signs $+$ and $-$, when the powers are different. For example, $a^3 + a^2$ is the sum of the second and third powers of a; and $a^5 - a^4$ is what remains when we subtract the fourth power of a from the fifth; and neither of these results can be abridged: but when we have powers of the same kind or degree, it is evidently unnecessary to connect them by signs; as $a^3 + a^3$ becomes $2a^3$, &c.

181. But in the *Multiplication* of powers, several circumstances require attention.

First, when it is required to multiply any power of a by a, we obtain the succeeding power; that is to say, the power whose exponent is greater by an unit. Thus, a^2, multiplied by a, produces a^3; and a^3, multiplied by a, produces a^4. In the same manner, when it is required to multiply by a the powers of any number represented by a, having negative exponents, we have only to add 1 to the exponent. Thus, a^{-1} multiplied by a produces a^0, or 1; which is made more evident by considering that a^{-1} is equal to $\frac{1}{a}$, and that the product of $\frac{1}{a}$ by a being $\frac{a}{a}$, it is consequently equal to 1; likewise a^{-2} multiplied by a, produces a^{-1}, or $\frac{1}{a}$; and

a^{-10} multiplied by a, gives a^{-9}, and so on. [See Art. 175, 176.]

182. Next, if it be required to multiply any power of a by a^2, or the second power, I say that the exponent becomes greater by 2. Thus, the product of a^2 by a^2 is a^4; that of a^2 by a^3 is a^5; that of a^4 by a^2 is a^6; and, more generally, a^n multiplied by a^2 makes a^{n+2}. With regard to negative exponents, we shall have a^1, or a, for the product of a^{-1} by a^2; for a^{-1} being equal to $\frac{1}{a}$, it is the same as if we had divided aa by a; consequently, the product required is $\frac{aa}{a}$, or a; also a^{-2}, multiplied by a^2, produces a^0, or 1; and a^{-3}, multiplied by a^2, produces a^{-1}.

183. It is no less evident, that to multiply any power of a by a^3, we must increase its exponent by three units; and that, consequently, the product of a^n by a^3 is a^{n+3}. And whenever it is required to multiply together two powers of a, the product will be also a power of a, and such that its exponent will be the sum of those of the two given powers. For example, a^4 multiplied by a^5 will make a^9, and a^{12} multiplied by a^7 will produce a^{19}, &c.

184. From these considerations we may easily determine the highest powers. To find, for instance, the twenty-fourth power of 2, I multiply the twelfth power by the twelfth power, because 2^{24} is equal to $2^{12} \times 2^{12}$. Now, we have already seen that 2^{12} is 4096; I say therefore that the number 16777216, or the product of 4096 by 4096, expresses the power required, namely, 2^{24}.

185. Let us now proceed to division. We shall remark, in the first place, that to divide a power of a by a, we must subtract 1 from the exponent, or diminish it by unity; thus, a^5 divided by a gives a^4; and a^0, or 1, divided by a, is equal to a^{-1} or $\frac{1}{a}$; also a^{-3} divided by a, gives a^{-4}.

186. If we have to divide a given power of a by a^2, we must diminish the exponent by 2; and if by a^3, we must subtract 3 units from the exponent of the power proposed; and, in general, whatever power of a it is required to divide by any other power of a, the rule is always to subtract the exponent of the second from the exponent of the first of those powers: thus a^{15} divided by a^7 will give a^8; a^6 divided by a^7 will give a^{-1}; and a^{-3} divided by a^4 will give a^{-7}.

187. From what has been said, it is easy to understand

the method of finding the powers of powers, this being done by multiplication. When we seek, for example, the square, or the second power of a^3, we find a^6; and in the same manner we find a^{12} for the third power, or the cube, of a^4. To obtain the square of a power, we have only to double its exponent; for its cube, we must triple the exponent; and so on. Thus, the square of a^n is a^{2n}; the cube of a^n is a^{3n}; the seventh power of a^n is a^{7n}, &c.

188. The square of a^2, or the square of the square of a, being a^4, we see why the fourth power is called the *biquadrate:* also, the square of a^3 being a^6, the sixth power has received the name of *the square-cubed.*

Lastly, the cube of a^3 being a^9, we call the ninth power the *cubo-cube:* after this, no other denominations of this kind have been introduced for powers; and, indeed, the two last are very little used.

CHAP. XVIII.

Of Roots, *with relation to* Powers *in general.*

189. Since the square root of a given number is a number, whose square is equal to that given number; and since the cube root of a given number is a number, whose cube is equal to that given number; it follows that any number whatever being given, we may always suppose such roots of it, that the fourth, or the fifth, or any other power of them, respectively, may be equal to the given number. To distinguish these different kinds of roots better, we shall call the square root, *the second root;* and the cube root, *the third root;* because according to this denomination we may call *the fourth root,* that whose biquadrate is equal to a given number; and *the fifth root,* that whose fifth power is equal to a given number, &c.

190. As the square, or second root, is marked by the sign $\sqrt{}$, and the cubic, or third root, by the sign $\sqrt[3]{}$, so the fourth root is represented by the sign $\sqrt[4]{}$; the fifth root by the sign $\sqrt[5]{}$; and so on. It is evident that, according to this method of expression, the sign of the square root ought to be $\sqrt[2]{}$: but as of all roots this occurs most frequently, it has been agreed, for the sake of brevity, to omit the number 2 as the sign of this root. So that when the radical sign has no num