the cube root of a product ab is found by multiplying together the cube roots of the factors. Hence, also, if we divide $\sqrt[3]{a}$ by $\sqrt[3]{b}$, the quotient will be $\sqrt[3]{\frac{a}{b}}$.

166. We farther perceive, that $2\sqrt[3]{a}$ is equal to $\sqrt[3]{8a}$, because 2 is equivalent to $\sqrt[3]{8}$; that $3\sqrt[3]{a}$ is equal to $\sqrt[3]{27a}$, $b\sqrt[3]{a}$ is equal to $\sqrt[3]{abbb}$; and, reciprocally, if the number under the radical sign has a factor which is a cube, we may make it disappear by placing its cube root before the sign; for example, instead of $\sqrt[3]{64a}$ we may write $4\sqrt[3]{a}$; and $5\sqrt[3]{a}$ instead of $\sqrt[3]{125a}$: hence $\sqrt[3]{16}$ is equal to $2\sqrt[3]{2}$, because 16 is equal to 8×2.

167. When a number proposed is negative, its cube root is not subject to the same difficulties that occurred in treating of square roots; for, since the cubes of negative numbers are negative, it follows that the cube roots of negative numbers are also negative; thus $\sqrt[3]{-8}$ is equal to -2, and $\sqrt[3]{-27}$ to -3. It follows also, that $\sqrt[3]{-12}$ is the same as $-\sqrt[3]{12}$, and that $\sqrt[3]{-a}$ may be expressed by $-\sqrt[3]{a}$. Whence we see that the sign $-$, when it is found after the sign of the cube root, might also have been placed before it. We are not therefore led here to impossible, or imaginary numbers, which happened in considering the square roots of negative numbers.

CHAP. XVI.

Of Powers *in general.*

168. The product which we obtain by multiplying a number once, or several times by itself, is called *a power*. Thus, a square which arises from the multiplication of a number by itself, and a cube which we obtain by multiplying a number twice by itself, are powers. We say also in the former case, that the number is raised to the second degree, or to the second power; and in the latter, that the number is raised to the third degree, or to the third power.

169. We distinguish those powers from one another by the number of times that the given number has been multiplied by itself. For example, a square is called the second

power, because a certain given number has been multiplied by itself; and if a number has been multiplied twice by itself we call the product the third power, which therefore means the same as the cube; also if we multiply a number three times by itself we obtain its fourth power, or what is commonly called the *biquadrate:* and thus it will be easy to understand what is meant by the fifth, sixth, seventh, &c. power of a number. I shall only add, that powers, after the fourth degree, cease to have any other but these numeral distinctions.

170. To illustrate this still better, we may observe, in the first place, that the powers of 1 remain always the same; because, whatever number of times we multiply 1 by itself, the product is found to be always 1. We shall therefore begin by representing the powers of 2 and of 3, which succeed each other as in the following order:

Powers.	Of the number 2.	Of the number 3.
1st	2	3
2d	4	9
3d	8	27
4th	16	81
5th	32	243
6th	64	729
7th	128	2187
8th	256	6561
9th	512	19683
10th	1024	59049
11th	2048	177147
12th	4096	531441
13th	8192	1594323
14th	16384	4782969
15th	32768	14348907
16th	65536	43046721
17th	131072	129140163
18th	262144	387420489

But the powers of the number 10 are the most remarkable: for on these powers the system of our arithmetic is founded. A few of them ranged in order, and beginning with the first power, are as follow:

1st	2d	3d	4th	5th	6th	
10,	100,	1000,	10000,	100000,	1000000,	&c.

171. In order to illustrate this subject, and to consider it in a more general manner, we may observe, that the

powers of any number, a, succeed each other in the following order:

1st 2d 3d 4th 5th 6th
a, aa, aaa, $aaaa$, $aaaaa$, $aaaaaa$, &c.

But we soon feel the inconvenience attending this manner of writing the powers, which consists in the necessity of repeating the same letter very often, to express high powers; and the reader also would have no less trouble, if he were obliged to count all the letters, to know what power is intended to be represented. The hundredth power, for example, could not be conveniently written in this manner; and it would be equally difficult to read it.

172. To avoid this inconvenience, a much more commodious method of expressing such powers has been devised, which, from its extensive use, deserves to be carefully explained. Thus, for example, to express the hundredth power, we simply write the number 100 above the quantity, whose hundredth power we would express, and a little towards the right-hand; thus a^{100} represents a raised to the 100th power, or the hundredth power of a. It must be observed, also, that the name *exponent* is given to the number written above that whose power, or degree, it represents, which, in the present instance, is 100.

173. In the same manner, a^2 signifies a raised to the 2d power, or the second power of a, which we represent sometimes also by aa, because both these expressions are written and understood with equal facility; but to express the cube, or the third power aaa, we write a^3, according to the rule, that we may occupy less room; so a^4 signifies the fourth, a^5 the fifth, and a^6 the sixth power of a.

174. In a word, the different powers of a will be represented by a, a^2, a^3, a^4, a^5, a^6, a^7, a^8, a^9, a^{10}, &c. Hence we see that in this manner we might very properly have written a^1 instead of a for the first term, to shew the order of the series more clearly. In fact, a^1 is no more than a, as this unit shews that the letter a is to be written only once. Such a series of powers is called also a geometrical progression, because each term is by one-time, or term, greater than the preceding.

175. As in this series of powers each term is found by multiplying the preceding term by a, which increases the exponent by 1; so when any term is given, we may also find the preceding term, if we divide by a, because this diminishes the exponent by 1. This shews that the term which precedes the first term a^1 must necessarily be

$\frac{a}{a}$, or 1; and, if we proceed according to the exponents, we immediately conclude, that the term which precedes the first must be a^0; and hence we deduce this remarkable property, that a^0 is always equal to 1, however great or small the value of the number a may be, and even when a is nothing; that is to say, a^0 is equal to 1.

176. We may also continue our series of powers in a retrograde order, and that in two different ways; first, by dividing always by a; and secondly, by diminishing the exponent by unity; and it is evident that, whether we follow the one or the other, the terms are still perfectly equal. This decreasing series is represented in both forms in the following Table, which must be read backwards, or from right to left.

	$\frac{1}{aaaaaa}$	$\frac{1}{aaaaa}$	$\frac{1}{aaaa}$	$\frac{1}{aaa}$	$\frac{1}{aa}$	$\frac{1}{a}$	1	a
1st.	$\frac{1}{a^6}$	$\frac{1}{a^5}$	$\frac{1}{a^4}$	$\frac{1}{a^3}$	$\frac{1}{a^2}$	$\frac{1}{a^1}$		
2d.	a^{-6}	a^{-5}	a^{-4}	a^{-3}	a^{-2}	a^{-1}	a^0	a^1

177. We are now come to the knowledge of powers whose exponents are negative, and are enabled to assign the precise value of those powers. Thus, from what has been said, it appears that

$$\left.\begin{array}{l} a^0 \\ a^{-1} \\ a^{-2} \\ a^{-3} \\ a^{-4} \end{array}\right\} \text{is equal to} \left\{\begin{array}{l} 1 \\ \frac{1}{a} \\ \frac{1}{aa} \text{ or } \frac{1}{a^2} \\ \frac{1}{a^3} \\ \frac{1}{a^4}, \text{ \&c.} \end{array}\right.$$

178. It will also be easy, from the foregoing notation, to find the powers of a product, ab; for they must evidently be ab, or a^1b^1, a^2b^2, a^3b^3, a^4b^4, a^5b^5, &c. and the powers of fractions will be found in the same manner; for example, those of $\frac{a}{b}$ are

$$\frac{a^1}{b^1}, \frac{a^2}{b^2}, \frac{a^3}{b^3}, \frac{a^4}{b^4}, \frac{a^5}{b^5}, \frac{a^6}{b^6}, \frac{a^7}{b^7}, \text{ \&c.}$$

179. Lastly, we have to consider the powers of negative numbers. Suppose the given number to be $-a$; then its powers will form the following series:

$$-a,\ +a^2,\ -a^3,\ +a^4,\ -a^5,\ +a^6,\ \&c,$$

Where we may observe, that those powers only become negative, whose exponents are odd numbers, and that, on the contrary, all the powers, which have an even number for the exponent, are positive. So that the third, fifth, seventh, ninth, &c. powers have all the sign $-$; and the second, fourth, sixth, eighth, &c. powers are affected by the sign $+$.

CHAP. XVII.

Of the Calculation *of* Powers.

180. We have nothing particular to observe with regard to the *Addition* and *Subtraction* of powers; for we only represent those operations by means of the signs $+$ and $-$, when the powers are different. For example, $a^3 + a^2$ is the sum of the second and third powers of a; and $a^5 - a^4$ is what remains when we subtract the fourth power of a from the fifth; and neither of these results can be abridged: but when we have powers of the same kind or degree, it is evidently unnecessary to connect them by signs; as $a^3 + a^3$ becomes $2a^3$, &c.

181. But in the *Multiplication* of powers, several circumstances require attention.

First, when it is required to multiply any power of a by a, we obtain the succeeding power; that is to say, the power whose exponent is greater by an unit. Thus, a^2, multiplied by a, produces a^3; and a^3, multiplied by a, produces a^4. In the same manner, when it is required to multiply by a the powers of any number represented by a, having negative exponents, we have only to add 1 to the exponent. Thus, a^{-1} multiplied by a produces a^0, or 1; which is made more evident by considering that a^{-1} is equal to $\frac{1}{a}$, and that the product of $\frac{1}{a}$ by a being $\frac{a}{a}$, it is consequently equal to 1; likewise a^{-2} multiplied by a, produces a^{-1}, or $\frac{1}{a}$; and