CHAP. XIV.

Of Cubic Numbers.

152. When a number has been multiplied twice by itself, or, which is the same thing, when the square of a number has been multiplied once more by that number, we obtain a product which is called a *cube*, or a *cubic number*. Thus, the cube of a is aaa, since it is the product obtained by multiplying a by itself, or by a, and that square aa again by a.

The cubes of the natural numbers, therefore, succeed each other in the following order *:

Numbers	1	2	3	4	5	6	7	8	9	10
Cubes	1	8	27	64	125	216	343	512	729	1000

153. If we consider the differences of those cubes, as we did of the squares, by subtracting each cube from that which comes after it, we obtain the following series of numbers:

7, 19, 37, 61, 91, 127, 169, 217, 271.

Where we do not at first observe any regularity in them; but if we take the respective differences of these numbers, we find the following series:

12, 18, 24, 30, 36, 42, 48, 54, 60;

in which the terms, it is evident, increase always by 6.

154. After the definition we have given of a cube, it will not be difficult to find the cubes of fractional numbers; thus, $\frac{1}{8}$ is the cube of $\frac{1}{2}$; $\frac{1}{27}$ is the cube of $\frac{1}{3}$; and $\frac{8}{27}$ is the cube of $\frac{2}{3}$. In the same manner, we have only to take the cube of the numerator and that of the denominator separately, and we shall have $\frac{27}{64}$ for the cube of $\frac{3}{4}$.

155. If it be required to find the cube of a mixed number, we must first reduce it to a single fraction, and then proceed in the manner that has been described. To find, for example, the cube of $1\frac{1}{2}$, we must take that of $\frac{3}{2}$, which

* We are indebted to a mathematician of the name of J. Paul Buchner, for Tables published at Nuremberg in 1701, in which are to be found the cubes, as well as the squares, of all numbers from 1 to 12000. F. T.

is $\frac{27}{8}$, or $3\frac{3}{8}$; also the cube of $1\frac{1}{4}$, or of the single fraction $\frac{5}{4}$, is $\frac{125}{64}$, or $1\frac{61}{64}$; and the cube of $3\frac{1}{4}$, or of $\frac{13}{4}$, is $\frac{2197}{64}$, or $34\frac{21}{64}$.

156. Since aaa is the cube of a, that of ab will be $aaabbb$; whence we see, that if a number has two or more factors, we may find its cube by multiplying together the cubes of those factors. For example, as 12 is equal to 3×4, we multiply the cube of 3, which is 27, by the cube of 4, which is 64, and we obtain 1728, the cube of 12; and farther, the cube of $2a$ is $8aaa$, and consequently 8 times greater than the cube of a: likewise, the cube of $3a$ is $27aaa$; that is to say, 27 times greater than the cube of a.

157. Let us attend here also to the signs $+$ and $-$. It is evident that the cube of a positive number $+a$ must also be positive, that is $+aaa$; but if it be required to cube a negative number $-a$, it is found by first taking the square, which is $+aa$, and then multiplying, according to the rule, this square by $-a$, which gives for the cube required $-aaa$. In this respect, therefore, it is not the same with cubic numbers as with squares, since the latter are always positive: whereas the cube of -1 is -1, that of -2 is -8, that of -3 is -27, and so on.

CHAP. XV.

Of Cube Roots, *and of* Irrational Numbers *resulting from them.*

158. As we can, in the manner already explained, find the cube of a given number, so, when a number is proposed, we may also reciprocally find a number, which, multiplied twice by itself, will produce that number. The number here sought is called, with relation to the other, *the cube root;* so that the cube root of a given number is the number whose cube is equal to that given number.

159. It is easy therefore to determine the cube root, when the number proposed is a real cube, such as in the examples in the last chapter; for we easily perceive that the cube root of 1 is 1; that of 8 is 2; that of 27 is 3; that of 64 is 4, and so on. And, in the same manner, the cube root of -27 is -3; and that of -125 is -5.

Farther, if the proposed number be a fraction, as $\frac{8}{27}$, the