2. Reduce $\frac{a}{b}$ and $\frac{a+b}{c}$ to a common denominator.

Ans. $\frac{ac}{bc}$ and $\frac{ab+b^2}{bc}$.

3. Reduce $\frac{3x}{2a}$, $\frac{2b}{3c}$, and d to fractions having a common denominator.

Ans. $\frac{9cx}{6ac}$, $\frac{4ab}{6ac}$ and $\frac{6acd}{6ac}$.

4. Reduce $\frac{3}{4}$, $\frac{2x}{3}$ and $a+\frac{2x}{a}$ to a common denominator.

Ans. $\frac{9a}{12a}$, $\frac{8ax}{12a}$, and $\frac{12a^2+24x}{12a}$.

5. Reduce $\frac{1}{2}$, $\frac{a^2}{3}$, and $\frac{x^2+a^2}{x+a}$, to a common denominator.

Ans. $\frac{3x+3a}{6x+6a}$, $\frac{2a^2x+2a^3}{6x+6a}$, $\frac{6x^2+6a^2}{6x+6a}$.

6. Reduce $\frac{b}{2a^2}$, $\frac{c}{2a}$, and $\frac{d}{a}$, to a common denominator.

Ans. $\frac{2a^2b}{4a^4}$, $\frac{2a^3c}{4a^4}$, and $\frac{4a^3d}{4a^4}$; or $\frac{b}{2a^2}$, $\frac{ac}{2a^2}$, and $\frac{2ad}{2a^2}$.

CHAP. X.

Of the Multiplication *and* Division *of* Fractions.

101. The rule for the multiplication of a fraction by an integer, or whole number, is to multiply the numerator only by the given number, and not to change the denominator: thus,

2 times, or twice $\frac{1}{2}$ makes $\frac{2}{2}$, or 1 integer;
2 times, or twice $\frac{1}{3}$ makes $\frac{2}{3}$; and
3 times, or thrice $\frac{1}{6}$ makes $\frac{3}{6}$, or $\frac{1}{2}$;
4 times $\frac{5}{12}$ makes $\frac{20}{12}$, or $1\frac{8}{12}$, or $1\frac{2}{3}$.

But, instead of this rule, we may use that of dividing the denominator by the given integer, which is preferable, when it can be done, because it shortens the operation. Let it be required, for example, to multiply $\frac{8}{9}$ by 3; if we multiply the numerator by the given integer we obtain $\frac{24}{9}$, which product we must reduce to $\frac{8}{3}$. But if we do not change the numerator, and divide the denominator by the integer, we find immediately $\frac{8}{3}$, or $2\frac{2}{3}$, for the given product; and, in the same manner, $\frac{13}{24}$ multiplied by 6 gives $\frac{13}{4}$, or $3\frac{1}{4}$.

102. In general, therefore, the product of the multiplication of a fraction $\frac{a}{b}$ by c is $\frac{ac}{b}$; and here it may be remarked, when the integer is exactly equal to the denominator, that the product must be equal to the numerator.

So that $\begin{cases} \frac{1}{2} \text{ taken twice, gives } 1; \\ \frac{2}{3} \text{ taken thrice, gives } 2; \\ \frac{3}{4} \text{ taken four times, gives } 3. \end{cases}$

And, in general, if we multiply the fraction $\frac{a}{b}$ by the number b, the product must be a, as we have already shewn; for since $\frac{a}{b}$ expresses the quotient resulting from the division of the dividend a by the divisor b, and because it has been demonstrated that the quotient multiplied by the divisor will give the dividend, it is evident that $\frac{a}{b}$ multiplied by b must produce a.

103. Having thus shewn how a fraction is to be multiplied by an integer; let us now consider also how a fraction is to be divided by an integer. This inquiry is necessary, before we proceed to the multiplication of fractions by fractions. It is evident, if we have to divide the fraction $\frac{2}{3}$ by 2, that the result must be $\frac{1}{3}$; and that the quotient of $\frac{6}{7}$ divided by 3 is $\frac{2}{7}$. The rule therefore is, to divide the numerator by the integer without changing the denominator. Thus:

$\frac{12}{25}$ divided by 2 gives $\frac{6}{25}$;
$\frac{12}{25}$ divided by 3 gives $\frac{4}{25}$; and
$\frac{12}{25}$ divided by 4 gives $\frac{3}{25}$; &c.

104. This rule may be easily practised, provided the numerator be divisible by the number proposed; but very often it is not: it must therefore be observed, that a fraction may be transformed into an infinite number of other expressions, and in that number there must be some, by which the numerator might be divided by the given integer. If it were required, for example, to divide $\frac{3}{4}$ by 2, we should change the fraction into $\frac{6}{8}$, and then dividing the numerator by 2, we should immediately have $\frac{3}{8}$ for the quotient sought.

In general, if it be proposed to divide the fraction $\frac{a}{b}$ by c, we change it into $\frac{ac}{bc}$, and then dividing the numerator ac by c, write $\frac{a}{bc}$ for the quotient sought.

105. When therefore a fraction $\frac{a}{b}$ is to be divided by an integer c, we have only to multiply the denominator by that number, and leave the numerator as it is. Thus $\frac{5}{8}$ divided by 3 gives $\frac{5}{24}$, and $\frac{9}{16}$ divided by 5 gives $\frac{9}{80}$.

This operation becomes easier, when the numerator itself is divisible by the integer, as we have supposed in article 103. For example, $\frac{9}{16}$ divided by 3 would give, according to our last rule, $\frac{9}{48}$; but by the first rule, which is applicable here, we obtain $\frac{3}{16}$, an expression equivalent to $\frac{9}{48}$, but more simple.

106. We shall now be able to understand how one fraction $\frac{a}{b}$ may be multiplied by another fraction $\frac{c}{d}$. For this purpose, we have only to consider that $\frac{c}{d}$ means that c is divided by d; and on this principle we shall first multiply the fraction $\frac{a}{b}$ by c, which produces the result $\frac{ac}{b}$; after which we shall divide by d, which gives $\frac{ac}{bd}$.

Hence the following rule for multiplying fractions. Multiply the numerators together for a numerator, and the denominators together for a denominator.

Thus $\frac{1}{2}$ by $\frac{2}{3}$ gives the product $\frac{2}{6}$, or $\frac{1}{3}$;
$\frac{2}{3}$ by $\frac{4}{5}$ makes $\frac{8}{15}$;
$\frac{3}{4}$ by $\frac{5}{12}$ produces $\frac{15}{48}$, or $\frac{5}{16}$; &c.

107. It now remains to shew how one fraction may be divided by another. Here we remark first, that if the two fractions have the same number for a denominator, the division takes place only with respect to the numerators; for it is evident, that $\frac{3}{12}$ are contained as many times in $\frac{9}{12}$ as 3 is contained in 9, that is to say, three times; and, in the same manner, in order to divide $\frac{8}{12}$ by $\frac{9}{12}$, we have only to divide 8 by 9, which gives $\frac{8}{9}$. We shall also have $\frac{6}{20}$ in $\frac{18}{20}$, 3 times; $\frac{7}{100}$ in $\frac{49}{100}$, 7 times; $\frac{7}{25}$ in $\frac{6}{25}$, $\frac{6}{7}$, &c.

108. But when the fractions have not equal denominators,

we must have recourse to the method already mentioned for reducing them to a common denominator. Let there be, for example, the fraction $\frac{a}{b}$ to be divided by the fraction $\frac{c}{d}$. We first reduce them to the same denominator, and there results $\frac{ad}{bd}$ to be divided by $\frac{cb}{db}$; it is now evident that the quotient must be represented simply by the division of ad by bc; which gives $\frac{ad}{bc}$.

Hence the following rule: Multiply the numerator of the dividend by the denominator of the divisor, and the denominator of the dividend by the numerator of the divisor; then the first product will be the numerator of the quotient, and the second will be its denominator.

109. Applying this rule to the division of $\frac{5}{8}$ by $\frac{2}{3}$, we shall have the quotient $\frac{15}{16}$; also the division of $\frac{3}{4}$ by $\frac{1}{2}$ will give $\frac{6}{4}$, or $\frac{3}{2}$, or $1\frac{1}{2}$; and $\frac{25}{48}$ by $\frac{5}{6}$ will give $\frac{150}{240}$, or $\frac{5}{8}$.

110. This rule for division is often expressed in a manner that is more easily remembered, as follows: Invert the terms of the divisor, so that the denominator may be in the place of the numerator, and the latter be written under the line; then multiply the fraction, which is the dividend by this inverted fraction, and the product will be the quotient sought. Thus, $\frac{3}{4}$ divided by $\frac{1}{2}$ is the same as $\frac{3}{4}$ multiplied by $\frac{2}{1}$, which makes $\frac{6}{4}$, or $1\frac{1}{2}$. Also $\frac{5}{8}$ divided by $\frac{2}{3}$ is the same as $\frac{5}{8}$ multiplied by $\frac{3}{2}$, which is $\frac{15}{16}$; or $\frac{25}{48}$ divided by $\frac{5}{6}$ gives the same as $\frac{25}{48}$ multiplied by $\frac{6}{5}$, the product of which is $\frac{150}{240}$, or $\frac{5}{8}$.

We see then, in general, that to divide by the fraction $\frac{1}{2}$ is the same as to multiply by $\frac{2}{1}$, or 2; and that dividing by $\frac{1}{3}$ amounts to multiplying by $\frac{3}{1}$, or by 3, &c.

111. The number 100 divided by $\frac{1}{2}$ will give 200; and 1000 divided by $\frac{1}{3}$ will give 3000. Farther, if it were required to divide 1 by $\frac{1}{1000}$, the quotient would be 1000; and dividing 1 by $\frac{1}{100000}$, the quotient is 100000. This enables us to conceive that, when any number is divided by 0, the result must be a number indefinitely great; for even the division of 1 by the small fraction $\frac{1}{1000000000}$ gives for the quotient the very great number 1000000000.

112. Every number, when divided by itself, producing unity, it is evident that a fraction divided by itself must also give 1 for the quotient; and the same follows from our rule: for, in order to divide $\frac{3}{4}$ by $\frac{3}{4}$, we must multiply $\frac{3}{4}$ by $\frac{4}{3}$, in

which case we obtain $\frac{12}{12}$, or 1; and if it be required to divide $\frac{a}{b}$ by $\frac{a}{b}$, we multiply $\frac{a}{b}$ by $\frac{b}{a}$; where the product $\frac{ab}{ab}$ is also equal to 1.

113. We have still to explain an expression which is frequently used. It may be asked, for example, what is the half of $\frac{3}{4}$? This means, that we must multiply $\frac{3}{4}$ by $\frac{1}{2}$. So likewise, if the value of $\frac{2}{3}$ of $\frac{5}{8}$ were required, we should multiply $\frac{5}{8}$ by $\frac{2}{3}$, which produces $\frac{10}{24}$; and $\frac{3}{4}$ of $\frac{9}{16}$ is the same as $\frac{9}{16}$ multiplied by $\frac{3}{4}$, which produces $\frac{27}{64}$.

114. Lastly, we must here observe, with respect to the signs $+$ and $-$, the same rules that we before laid down for integers. Thus $+\frac{1}{2}$ multiplied by $-\frac{1}{3}$, makes $-\frac{1}{6}$; and $-\frac{2}{3}$ multiplied by $-\frac{4}{5}$, gives $+\frac{8}{15}$. Farther $-\frac{5}{8}$ divided by $+\frac{2}{3}$, gives $-\frac{15}{16}$; and $-\frac{3}{4}$ divided by $-\frac{3}{4}$, gives $+\frac{12}{12}$, or $+1$.

QUESTIONS FOR PRACTICE.

1. Required the product of $\frac{x}{6}$ and $\frac{2x}{9}$. *Ans.* $\frac{x^2}{27}$.

2. Required the product of $\frac{x}{2}$, $\frac{4x}{5}$, and $\frac{10x}{21}$. *Ans.* $\frac{4x^3}{21}$.

3. Required the product of $\frac{x}{a}$ and $\frac{x+a}{a+c}$. *Ans.* $\frac{x^2+ax}{a^2+ac}$.

4. Required the product of $\frac{3x}{2}$ and $\frac{3a}{b}$. *Ans.* $\frac{9ax}{2b}$.

5. Required the product of $\frac{2x}{5}$ and $\frac{3x^2}{2a}$ *Ans.* $\frac{3x^3}{5a}$.

6. Required the product of $\frac{2x}{a}$, $\frac{3ab}{c}$, and $\frac{3ac}{2b}$. *Ans.* $9ax$.

7. Required the product of $b+\frac{bx}{a}$ and $\frac{a}{x}$.

Ans. $\frac{ab+bx}{x}$.

8. Required the product of $\frac{x^2-b^2}{bc}$ and $\frac{x^2+b^2}{b+c}$.

Ans. $\frac{x^4-b^4}{b^2c+bc^2}$

9. Required the product of x, $\frac{x+1}{a}$, and $\frac{x-1}{a+b}$.

Ans. $\frac{x^3-x}{a^2+ab}$.

10. Required the quotient of $\frac{x}{3}$ divided by $\frac{2x}{9}$. *Ans.* $1\frac{1}{2}$.

11. Required the quotient of $\frac{2a}{b}$ divided by $\frac{4c}{d}$.

Ans. $\frac{ad}{2bc}$.

12. Required the quotient of $\frac{x+a}{2x-2b}$ divided by $\frac{x+b}{5x+a}$.

Ans. $\frac{5x^2+6ax+a^2}{2x^2-2b^2}$.

13. Required the quotient of $\frac{2x^2}{a^3+x^3}$ divided by $\frac{x}{x+a}$.

Ans. $\frac{2x^2+2ax}{x^3+a^3}$.

14. Required the quotient of $\frac{7x}{5}$ divided by $\frac{12}{13}$. *Ans.* $\frac{91x}{60}$.

15. Required the quotient of $\frac{4x^2}{7}$ divided by $5x$. *Ans.* $\frac{4x}{35}$.

16. Required the quotient of $\frac{x+1}{6}$ divided by $\frac{2x}{3}$.

Ans. $\frac{x+1}{4x}$.

17. Required the quotient of $\frac{x-b}{8cd}$ divided by $\frac{3cx}{4d}$.

Ans. $\frac{x-b}{6c^2x}$.

18. Required the quotient of $\frac{x^4-b^4}{x^2-2bx+b^2}$ divided by - - $\frac{x^2+bx}{x-b}$.

Ans. $x+\frac{b^2}{x}$.